On the Changing Relationship between Net Public Foreign Assets and Growth*

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Abstract

This paper documents novel stylized facts and illustrates a simple mechanism explaining patterns of net public foreign assets across countries and time. Previous literature found an unexpected negative correlation between growth and net public foreign assets from 1980 to the mid-2000s. Analyzing data up to 2019 we find that this result no longer holds. We document a significant reversal since 2004, with the correlation now zero or weakly positive. Empirically, we attribute this shift to a substantial substitution from public debt towards international reserves, particularly for slower-growing countries. Simultaneously, low-growth countries experienced heightened productivity volatility. Augmenting an open economy neoclassical growth model to include uncertainty, we demonstrate that this increased risk faced by low-growth economies explains 46% of the change in correlation.

JEL Codes: F21, F32, F41, F43, F44, O47

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1 Introduction

Net public foreign asset positions of emerging and low-income countries vary significantly across countries and across time. This paper takes a new look at the patterns of net public foreign assets in emerging market and low-income developing countries. We document novel stylized facts showing a stark reversal in the correlation between productivity growth rates and net public foreign asset flows. We explore this changing relationship over the 1980 to 2019 period, and answer the question: "Why did the correlation between net public foreign assets and growth change?"

The neoclassical growth model predicts that productivity growth rates should determine capital flows across countries. The theory predicts that foreign asset inflows and growth are positively correlated. In the years prior to 2004 there was a significant discrepancy between the predictions of the neoclassical growth model and flows of capital across emerging and low-income developing countries observed in the data (first noted by Lucas (1990)). The Lucas puzzle showed that the volume of capital flows from rich to poor countries were quite small. In later work, Gourinchas and Jeanne (2013) showed that net foreign assets and growth were negatively correlated across countries, they dubbed this the "allocation puzzle". We find that this result no longer holds.[1] We uncover a stark reversal since 2004, with the correlation now zero or weakly positive.

We show that both the relationship between public external debt and growth, and the relationship between international reserves and growth changed after the year 2004. Moreover, we show that the relationship between public external debt from private creditors and growth has not changed, while the relationship with public external debt by official creditors has. We show that these results hold across an array of robustness checks.

Mechanically, the changing correlation was caused by a sharp substitution away from debt, towards international reserves, for low growth countries. We show that low growth countries simultaneously experienced a pronounced increase in aggregate risk. In this paper, we argue that increasing volatility is enough to offset the phenomenon of the so-called allocation puzzle. Hence, we suggest the simple explanatory mechanism of an increased motive for precautionary savings.

We explore a myriad of alternative explanations that could possibly account for these changes: (1) the heavily indebted poor countries initiative, (2) commodity exporters, (3) central bank independence, (4) market exclusion, (5) capital mobility, and (6) flows of funds from China. We conclude that no other explanations can account for the observed change.

[1] Refer to *Figure C.1* in Appendix C to see the reversal in the relationship between total net foreign assets and growth. We document the relationship between net public foreign assets and growth in much detail throughout the remainder of the paper.

Quantitatively, we provide additional evidence in support of our mechanism. We build a stochastic, open-economy, neoclassical growth model that can account for the reversal in correlation between net public foreign assets and growth. Using wedges on capital and savings, the model is calibrated to match net public inflows and average investment rates in the data. We perform a counterfactual exercise where we decompose the contribution of changing TFP volatility vs. other forces present in the model. The quantitative illustrations show that the increase in volatility of the stochastic TFP accounts for 46% of the observed change in the correlation between net public foreign assets and growth.

This paper relates to a literature that analyzes the relationship between net foreign assets position and growth (Gourinchas and Jeanne, 2013; Aguiar and Amador, 2011; Alfaro et al., 2014). Gourinchas and Jeanne (2013) find that there is a negative correlation between net foreign asset positions and growth.[2] The authors argue that this negative relationship can be explained by savings and capital wedges that act as taxes to capital flows. Following Gourinchas and Jeanne (2013), Aguiar and Amador (2011) show that this negative correlation is driven by net public foreign assets. More specifically, these authors find that the relationship is largely a product of international reserve accumulation. These two papers analyze the relationship between growth and net foreign assets for a specific point in time. We show how this relationship has changed over time and offer an explanation as to why it has changed.

Alfaro et al. (2014) study the same question but focus on a more detailed breakdown of public external debt. In particular, they look at different types of creditors in order to understand how different components of public external debt behave. In their paper, they consider real GDP growth rates relative to the US. They show that using the definition in Gourinchas and Jeanne (2013), (productivity "catch-up" relative to the U.S.) is equivalent to growth rates.[3] In this paper we will make use of the growth measure discussed in Aguiar and Amador (2011) which Alfaro et al. (2014) showed is equivalent to the measure used in Gourinchas and Jeanne (2013).

The remainder of the paper is organized as follows. *Section 2* describes the data used for the empirical analysis. *Section 3* presents our main empirical analysis and findings. In *Section 4* we explore alternative explanations to our main finding. *Section 5.1* outlines the model and *Section 5.2* the calibration procedure. In *Section 5.3* we conduct our main decomposition exercise. *Section 6* concludes.

[2] Figure B.3 in the Appendix B.2 shows *Figure 1* from Gourinchas and Jeanne (2013) replicated for this paper, using our updated data and the provided replication package.

[3] The productivity catch-up for each country in Gourinchas and Jeanne (2013) is defined as $\bar{A}_{2000}/(g^*\bar{A}_{2000}) - 1$, where $\bar{A}$ is the value of the Hodrick–Prescott trend component of productivity estimate A_t and g^* is the average annual TFP growth for the United states between 1980 and 2000.

2 Data and Sample Selection

In this section we briefly describe the data sources for our main empirical analysis in *Section 3*. The data used for the empirical analysis in this paper originates from several sources. For all variables relating to public external debt, data is obtained from the International Debt Statistics (IDS) which is compiled by the World Bank. The IDS reports all external public and publicly guaranteed long-term debt for emerging and low-income countries. It also reports the public and publicly guaranteed debt by type of creditor. That is, if the creditor is an official creditor (multilateral and bilateral lenders) or a private creditor (commercial banks, bonds, and other). From this dataset we use the overall debt stocks reported and the breakdowns by type of creditor.

The international reserves data comes from the International Financial Statistics (IFS) administered by the International Monetary Fund (IMF). The data for total factor productivity (TFP) and capital stocks comes from the Penn World Table.[4] Investment data comes from the World Development Indicators (WDI) from the World Bank. Data on GDP per capita is also obtained from the WDI.

Our sample period covers 1980 to 2019. We end the sample in 2019 before the onset of the COVID-19 crisis, given that during the pandemic several measures were implemented by the World Bank and the IMF to mitigate the negative impacts of the pandemic. These measures included emergency financing, grants for debt relief, bilateral debt reliefs, or additional SDR allocations. All these policies may have temporarily changed the dynamics of public flow of assets; hence we exclude them from our analysis.

The sample of countries used is emerging and low-income countries; see *Table C.1* in Appendix C for a full list of countries. Our sample of countries is nearly identical to that of Gourinchas and Jeanne (2013) and Aguiar and Amador (2011). Appendix B shows that our results are robust to the sample of countries used. Appendix A contains additional details on the underlying data. Additional data sets are introduced for the analysis in *Section 4*.

3 Relationship between Public NFA and Growth

This paper examines the relationship between public net foreign assets and economic growth. We focus on this aspect rather than total net foreign assets as previous research has identified public net foreign assets as the key factor driving the link between total net foreign assets and growth. See *Figure C.1* in Appendix C to see changes in the correlation between *total*

[4] Feenstra, Robert C., Robert Inklaar and Marcel P. Timmer (2015), "The Next Generation of the Penn World Table" American Economic Review, 105(10), 3150-3182, available for download at www.ggdc.net/pwt

NFA position and growth overtime.

The net public foreign asset position of a country is defined as the difference between public external debt and international reserves. That is, we define,

$$\text{PuNFA}_{j,T} = \frac{\text{US\$ External PPG Debt}_{j,T}}{\text{US\$ GDP}_{j,T}} - \frac{\text{US\$ International Reserves}_{j,T}}{\text{US\$ GDP}_{j,T}} \quad (1)$$

where j denotes an individual country and T represents a given year. PuNFA is the net public foreign asset position, External PPG Debt is the external public and publicly guaranteed debt, and International Reserves is international reserves minus gold.

Following Aguiar and Amador (2011) we investigate the relationship between changes in net public foreign assets and average growth rate (relative to the United States) of a country. We define changes in the net public foreign asset position as,

$$\Delta\text{PuNFA}_{j,T} = \frac{\text{PuNFA}_{j,T} - \text{PuNFA}_{j,1980}}{T - 1980} \quad (2)$$

where $\Delta\text{PuNFA}_{j,T}$ is the flow of public foreign assets to country j in year T, relative to 1980. That is, $\Delta\text{PuNFA}_{j,T}$ represents the change in PuNFA over the 1980 to T time period. The growth rate of a country is determined as changes in real GDP per-capita. We define the average growth rate of a country relative to the U.S. as,

$$\Delta\text{g}_{j,T} = \frac{\sum_{k=1980}^{T} g_{j,k}}{T - 1980} - \frac{\sum_{k=1980}^{T} g_{US,k}}{T - 1980} \quad (3)$$

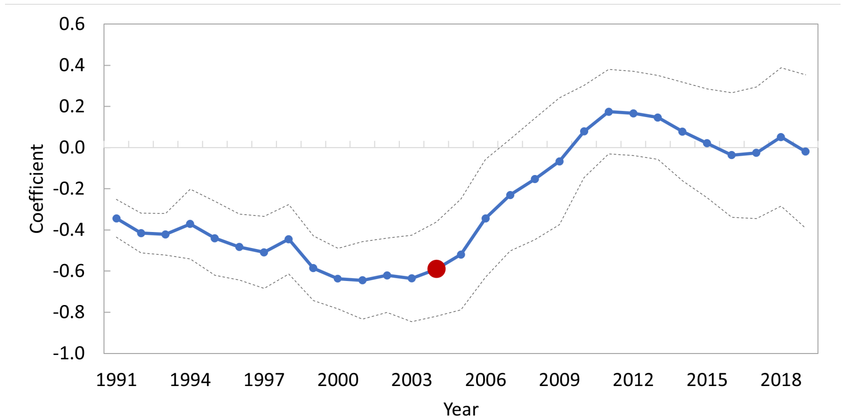
where $g_{j,k}$ is real GDP per capita growth of country j at time k, and $g_{US,k}$ is real per capita GDP growth of the US at time k. Hence, the resulting $\Delta\text{g}_{j,T}$ is the average GDP per capita growth over the 1980 to T time period relative to the average growth rate of the US over the same time period. To illustrate how the relationship has changed over time we estimate the following regression, for $T \in \{1991, 1992, ..., 2019\}$,

$$\Delta\text{g}_{j,T} = \alpha + \beta_T \Delta\text{PuNFA}_{j,T} + \epsilon_{j,T} \quad (4)$$

That is, we begin by determining the correlation between changes in growth and changes in PuNFA using the 1980-1991 time period and running the above described regression. This regression estimates the coefficient β_{1991}. We then re-run the same regression for the 1980-1992 time period, and so on, up until the 1980-2019 period. This produces a sequence of the coefficients of interest β_T. *Figure 1* shows how this coefficient has evolved over time.

We start the regression in 1991 which allows for a sufficient time horizon to mitigate the effects of business cycles. *Figure C.3* in Appendix C shows the results for the regressions

Figure 1: Relationship between Net Public Foreign Assets and Growth Over Time



Notes: This figure plots the resulting coefficient β from equation (4). The coefficient is the blue line and the gray lines are a 90 percent confidence interval.

when the base year is 1970. The same reversal is observed in the mid-2000s.

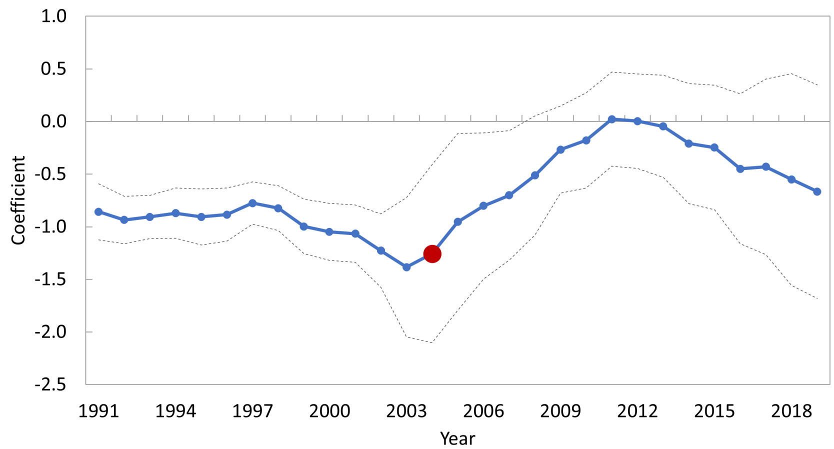
Beginning in 2004, the relationship between growth and net public foreign assets changed. A red dot highlights this year. Before the year 2004 the coefficient was close to -0.5. Starting in 2004 the correlation becomes zero for the year 2008 and even positive for some years (2009-2014), before going back to zero again. The string of papers (Aguiar and Amador, 2011; Alfaro et al., 2014; Gourinchas and Jeanne, 2013) that studied this relationship ended their sample in the early 2000s, immediately prior to the onset of this changing trend.[5] In Appendix C we show that the relationship is robust to removing outliers that can drive the correlation (*Figure C.4*), doing a 20 year rolling regression (*Figure C.5*), and calculating growth rates using geometric means instead of arithmetic means (*Figure C.6*).

Net public foreign assets have two components: public external debt and international reserves. We now investigate the importance of these two components in explaining this changing relationship. First, we run the same regression as in equation (4), however the dependent variable is now the negative of the change in international reserves.

Figure 2 illustrates the relationship between international reserves and growth over time. Consistent with past findings in this literature, our analysis shows a robust correlation between international reserves and the broader category of net public foreign assets and growth. The second component of the net public foreign asset position is external public and publicly guaranteed debt. *Figure 3* presents the regression results with public external debt as the independent variable. For both reserves and public external debt, the negative relationship

[5] To limit the scope of this paper we have focused on the flow of public funds. However, Appendix C documents a reversal in trend of private flows and total flows. This observation is also a novel addition to the literature. We believe an investigation of these two findings are an important avenue for future research.

Figure 2: Relationship between (Negative) International Reserves and Growth Over Time



Notes: This figure plots the resulting coefficient β from equation (4) when the independent variable is the negative of international reserves. The coefficient is the blue line, and the gray lines are a 90 percent confidence interval.

also began to starkly change during the mid-2000s. The coefficient associated with international reserves is of a significantly greater magnitude compared to that of external debt. This suggests that the overall correlation is primarily driven by international reserves.

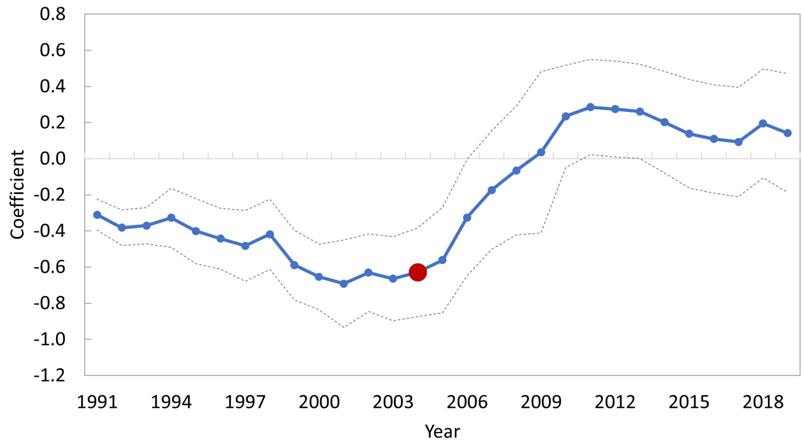
Following Alfaro et al. (2014) we explore how the relationship differs by type of creditors for the public external debt. Using their methodology, we estimate equation (4), using the change in external debt from official creditors (bilateral and multilateral) as the dependent variable. The results, shown in *Figure 4*, demonstrate that the relationship between this type of debt has also changed over time. The correlation has become zero since 2007 when the sample in Alfaro et al. (2014) ended.

The last component we analyze is public external debt from private creditors (banks, bonds, and others). The regression coefficients from equation (4) are plotted in *Figure 5*. We find that the correlation between public external debt from private creditors is positively correlated with growth. This relationship has not changed over time and remains positive. This suggests that public debt originating from private creditors behaves according to the dynamics of the neoclassical growth model.

Given our documentation of a changing relationship between net public foreign assets and growth, we now turn to our main research question: "Why has the correlation between net public foreign assets and growth changed?" *Figures 2* and *3* indicate the dynamics are influenced by the relationship between both international reserves, public external debt, and growth.

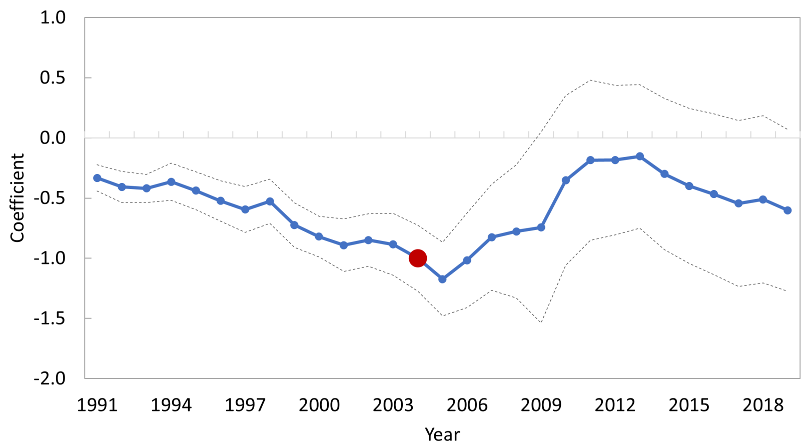
The trend in *Figure 2* indicates that countries with higher growth rates (relative to the U.S.) previously accumulated more reserves (or savings) compared to their counterparts with

Figure 3: Relationship between Public External Debt and Growth Over Time



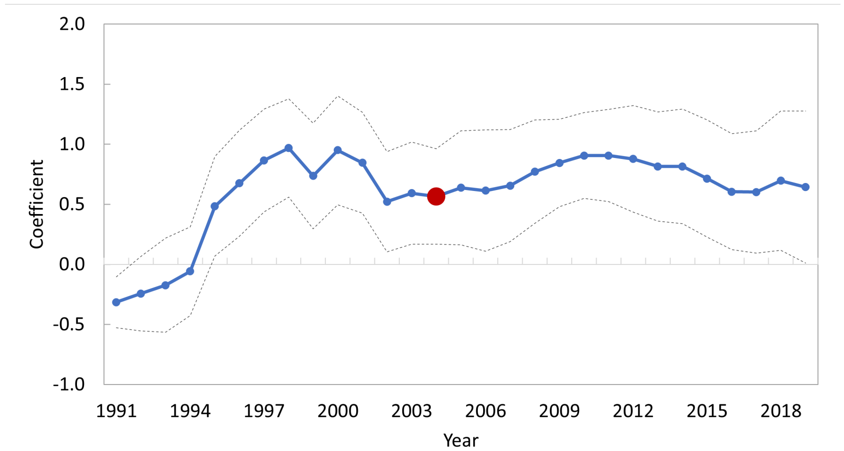
Notes: This figure plots the resulting coefficient β from equation (4) when the independent variable is public external debt. The coefficient is the blue line and the gray lines are a 90 percent confidence interval.

Figure 4: Relationship between Public External Debt (Official Creditors) and Growth Over Time



Notes: This figure plots the resulting coefficient β from equation (4) when the independent variable is public external debt from official creditors. The coefficient is the blue line, and the gray lines are a 90 percent confidence interval.

Figure 5: Relationship between Public External Debt (Private Creditors) and Growth Over Time



Notes: This figure plots the resulting coefficient β from equation (4) when the independent variable is public external debt from private creditors. The coefficient is the blue line, and the gray lines are a 90 percent confidence interval.

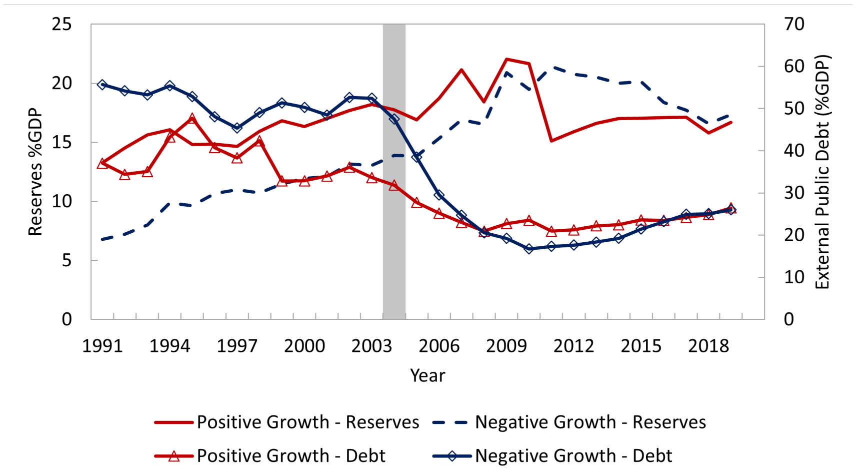
lower growth rates. *Figure 2* also shows, for the pre-2004 period, increases in international reserves (recall we take the negative of reserves) are negatively correlated with growth. That is, on average, higher growth countries exhibit faster reserve accumulation. However, for the post-2004 period we see the opposite correlation. This suggests that mechanically, at least one of two things is occurring. Either countries with higher growth rates have reduced their savings, or those with lower growth rates increased their savings, or a combination of both.

To understand how the accumulation of international reserves changed across time and countries, we categorize countries into two groups. We call negative growth (positive growth) countries those with a growth rate lower (higher) than that of the U.S. for any given year. *Figure 6* plots the average level of international reserves to GDP and the external debt to GDP across these two sets of countries.

Refer to Appendix C for figures when the sample of negative growth countries are those that had negative growth rates compared to the U.S. in the year 2000 and the sample of positive countries are those that had positive growth rates compared to the U.S. in the year 2000. The overall trends remain consistent when we use the "constant" sample. However, it is important to note, we are simply providing intuition as to why the correlation from our regression result must mechanically change. The regression analysis (and any correlation) will always have countries coming in and out of the high growth groupings.

The results in *Figure 6* show that before 2004, positive growth countries accumulated on average more international reserves compared to negative growth countries. Beginning in 2004, negative growth countries began to accumulate more reserves and by 2011 they were

Figure 6: International Reserves and Public External Debt to GDP (simple average)



Source: IDS World Bank, WDI World Bank, BOP IMF, and authors' calculations.

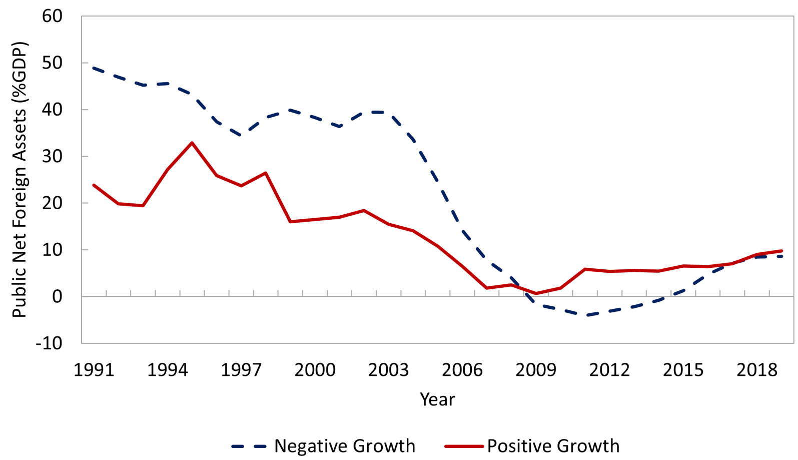
(on average) accumulating more reserves than positive growth countries. The opposite relationship is observed for public external debt. Low growth countries accumulated on average higher debt levels in the period prior to 2004 compared to high growth countries. Again, beginning in 2004 negative growth countries began reducing their debt levels significantly, going from an average of 53% of GDP to 17% of GDP by the year 2011. The substitution observed between international reserve accumulation and public external debt in negative growth countries is what mechanically drives the change in correlation observed between net public foreign assets and growth.

The overall net public foreign asset position for both types of countries is depicted in *Figure 7*. After 2004, negative growth countries decreased their net public foreign asset position significantly while positive growth countries saw only a small reduction in their net positions.

3.1 Volatility

The decrease in public external debt and increase in international reserves by low growth countries underlies the correlation change between growth and net public foreign asset position during the post 2004 period. We now aim to determine what explains this pronounced substitution of debt to reserves for low growth countries. We argue that the reason why low growth countries changed the way they accumulated foreign assets was the result of a sudden increase in productivity volatility. In *Section 4* we explore an array of other possible explanations and show that none can account for observed changes in the correlation between net public foreign assets and growth.

Figure 7: Net Public Foreign Assets to GDP (simple average)



Source: IDS World Bank, WDI World Bank, BOP IMF, and author's calculations.

Using data from the Penn World Tables and closely following the methodology outlined by Caselli and Feyrer (2007), we calculate total factor productivity (TFP) for each country in our sample.[6] The calculated TFP is demeaned, and using a 5-year backward rolling window we obtain the standard deviation of each country. The averages across our two sets of countries are plotted in *Figure 8*.

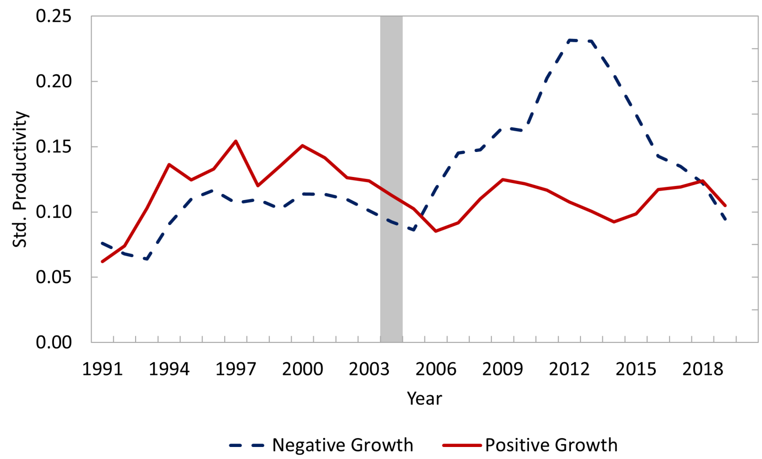
Prior to 2004, both groups of countries faced similar volatility in their TFP levels. However, post-2004, the set of negative growth countries witnessed a significant increase in TFP volatility, which translates into higher levels of risk or uncertainty for these countries. This rise in volatility is directly correlated with the increased accumulation of international reserves (correlation coefficient of 0.7) and the reduction in external debt levels (correlation coefficient of -0.9) observed in these countries. This suggests that countries exhibited higher precautionary savings motives in response to heightened uncertainty. In *Section 5.1* we quantitatively evaluate this mechanism using a simple open economy neoclassical growth model. In our counterfactual economy with higher TFP volatility we find further evidence in support of this mechanism.

4 Alternative Explanations

In this section we highlight additional explanations for the change in correlation between net public foreign asset position and growth. We conclude that none of the other mechanisms can explain the changing correlation.

[6] See *Figure C.7* in Appendix C for a robustness exercises where we perform the same analysis using the marginal product of capital.

Figure 8: TFP Volatility (simple average)



Source: PennWorld Table and authors' calculations.

4.1 Heavily Indebted Poor Countries Initiative

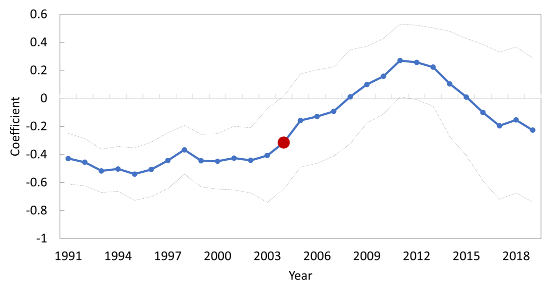
In 1996 the IMF and the World Bank launched the Heavily Indebted Poor Countries (HIPC) initiative to ensure that no low-income country faced an unmanageable debt burden. The aim of the program was to provide debt relief to a select group of countries meeting a set of strict criteria. Over 30 countries participated in the initiative and reduced their debt levels significantly. Given that the HIPC initiative was targeted to low-income developing countries, the reduction in debt levels seen in the late 1990s and early 2000s (as a consequence of the HIPC initiative) could potentially explain the change in the relationship between net public foreign asset position and growth observed in *Figure 1*. To test this hypothesis, equation (4) is estimated excluding countries that participated in the HIPC initiative.[7] The results suggests that countries that participated in the HIPC initiative cannot (by itself) explain the change in the trend observed in the data.

Figure 9a shows the regressions results when estimated only on non-HIPC participants. The overall trend, with a reversal in the correlation coefficient between growth and the net public foreign asset position, is still evident. Moreover, we computed average international reserve accumulation and public external debt by the two set of countries (negative and positive) when the HIPC initiative participants are excluded from the sample. *Figure 9b* shows that the increase in international reserves accumulation for the negative growth countries is still present even without the HIPC countries. A similar trend in public external debt is observed. Without the inclusion of HIPC countries, the drop in the levels of public external

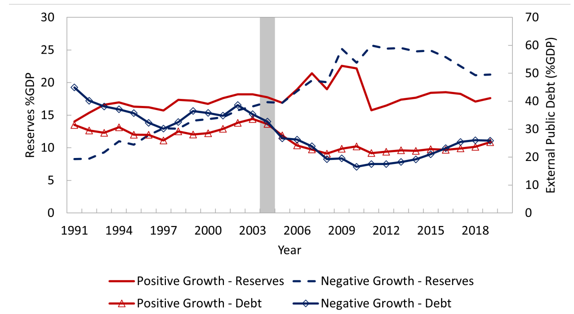
[7] *Table C.2* list the countries in our sample that participated in the HIPC initiative.

Figure 9: Net Public Foreign Assets excluding HIPC Participants

(a) Relationship between Net Public Foreign Assets and Growth Over Time Excluding HIPC participants



(b) International Reserves and Public External Debt to GDP Excluding HIPC Participants (simple average)



Notes: This figure plots the resulting coefficient β excluding countries that participated in the HIPC initiative from equation (4). The coefficient is the blue line, and the gray lines are a 90 percent confidence interval. *Source:* IDS World Bank, WDI World Bank, BOP IMF and authors' calculations.

debt is smaller compared to those in *Figure 6*. Nevertheless, it appears that the HIPC initiative is not the primary driver of the reversal in the correlation between net public foreign asset position and growth.

4.2 Commodity Exporters

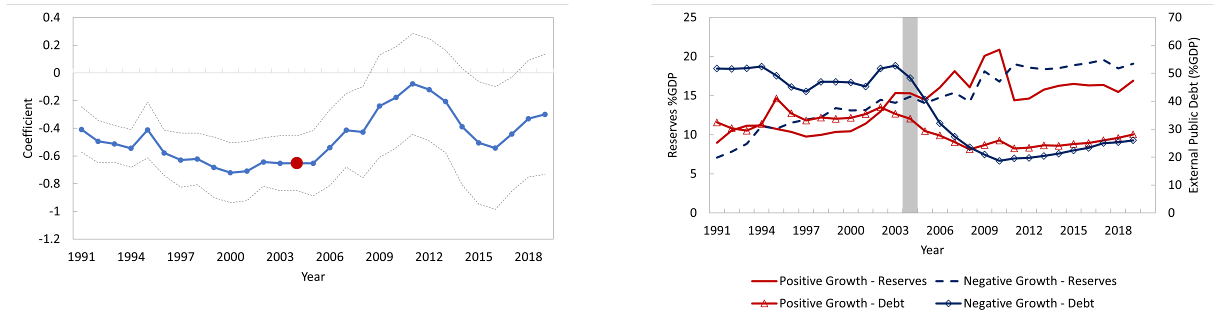
The early 2000s were marked by a commodity boom (Carter et al., 2011) and the integration of China into the WTO. Both had sizable impacts on commodity exporters and their balances of trade. These events could potentially influence the net foreign asset inflows and growth rates of countries. Given that the correlation depends both on growth rates as well as net inflows of countries, the difference could be explained by changes in the growth rates of commodity exporting countries. According to Carter et al. (2011), real income growth for emerging and developing countries grew on average 7.2% per year during this period. This may potentially account for the observed trend reversal in the correlation between net public foreign assets and growth. *Figures 10a and 10b* indicate that countries that are commodity exporters do not exclusively explain the overall reversal in the correlation between these two variables. Specifically, *Figure 10a* shows that the change in correlation persists for countries that are not commodity exporters.

Additionally, in *Figure 10b*, the averages for non-commodity exporters reveal that international reserves for countries with negative growth still increased, while their external debt decreased. Overall, non-commodity exporters which faced lower growth rates than the U.S. decreased their net public foreign asset position by a larger magnitude than countries which faced positive growth rates. Therefore, countries that are commodity exporters cannot

Figure 10: Net Public Foreign Assets excluding Commodity Exporters

(a) Relationship between Net Public Foreign Assets and Growth Over Time Excluding Commodity Exporters

(b) International Reserves and Public External Debt to GDP excluding Commodity Exporters (simple average)



Notes: This figure plots the resulting coefficient β excluding countries that are commodity exporters from equation (4). The coefficient is the blue line, and the gray lines are a 90 percent confidence interval. *Source:* IDS World Bank, WDI World Bank, BOP IMF and authors' calculations.

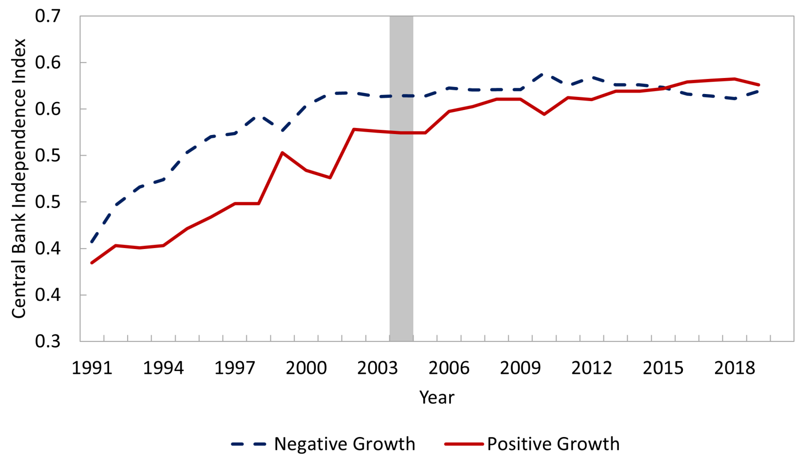
adequately explain the change in the relationship observed in the data.

4.3 Central Bank Independence

Throughout the 1990s several emerging market and developing economies implemented market oriented economic reforms, often referred to as the "Washington Consensus". Some of these policies consisted of trade liberalization, competitive exchange rates, or liberalization of inward foreign direct investment. A plausible explanation for the reversal in correlation between net public foreign asset and growth is that countries which implemented these types of policies experienced a significant increase in their international reserves. This surge may be attributed to the growing central bank independence or changes in exchange rate regimes. Suppose, on average, either the negative growth or positive growth countries had different central bank practices, such as different exchange rate regimes or whether the central bank can lend to the government in the primary market. This could explain the reversal in observed correlation, given that the central bank controls international reserves. As shown in *Figure 2* international reserves are a key determinant in the overall observed correlation.

Using the central bank independence index constructed by Garriga (2016) and updating to 2019, *Figure 11* shows the index over time for our two sets of countries. The plot shows that there is not a sudden change in the index that could potentially account for the observed increase in international reserves accumulation. Additionally, using data from the IMF Annual Report on Exchange Arrangements and Exchange Restrictions (AREAER) dataset, we classify a country as having a floating exchange rate regime if it is listed as either "floating" or "free floating". Given the changes in reserve accumulation, there could be systematic

Figure 11: Central Bank Independence Index (simple average)



Source: Garriga (2016) and authors' calculations.

differences in exchange rate regimes between the two sets of countries which may explain the changing relationship.

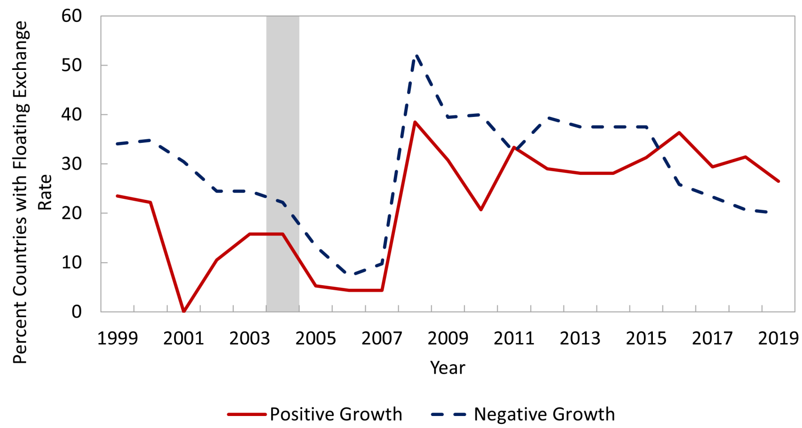
Figure 12 shows the percentage of countries in our sample with a floating exchange rate regime over time, for both positive and negative growth countries. The sustained increase in international reserve accumulation and the sudden increase observed for the negative growth countries after 2004, seems unrelated to exchange rate management. On average, both sets of countries have a similar composition of exchange rate regimes. By 2004, the percent of countries with a floating exchange rate was 20% for the negative growth countries and 15% for the positive growth countries. In 2008, the percent for both sets of countries increased to 52% and 40%, respectively. Hence, both sets of countries demonstrated, on average, similar changes in their exchange rate regimes. The data does not reveal a discernible pattern in the number of countries with a floating exchange rate that could help us explain the observed patterns.

4.4 Market Exclusion

The 1980s and 1990s were characterized by many countries facing sovereign debt crises. Given the rise in international reserve accumulation observed in the data, one explanation could be that countries were excluded from financial markets and had no access to external debt. In the sovereign debt literature (Arellano, 2008; Chatterjee and Eyigungor, 2012; Aguiar and Amador, 2021), it is assumed that a country has no access to financial markets as long as it is in a default state.

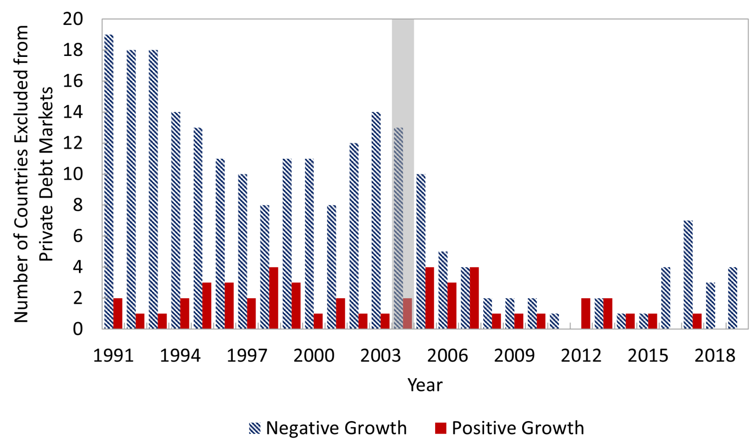
Using data from Asonuma and Trebesch (2016), and Asonuma et al. (2017), *Figure 13*

Figure 12: Percent of Countries with Free Floating Exchange Rates (simple average)



Source: IMF AREAER Database and authors' calculations.

Figure 13: Number of Countries in a Default with the Private Sector

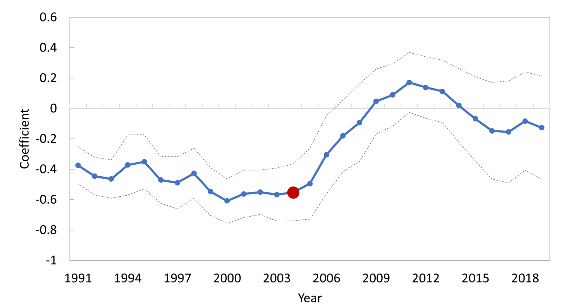


Source: Asonuma and Trebesch (2016), Asonuma et al. (2017), and authors' calculations.

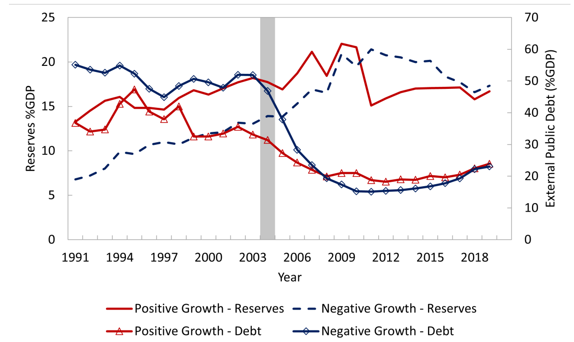
shows the number of countries in a default episode with private creditors over time. Despite a substantial number of countries experiencing default episodes in the 1990s, there is a sharp decline after 2004. This decrease is associated with the HIPC initiative, as some default episodes were resolved through the program. However, after 2004 there is no sudden increase in default episodes by either the negative growth or the positive growth countries that could potentially explain the change in the relationship between net public foreign asset position and growth.

Figure 14: Net Public Foreign Assets excluding Flows from China

(a) Relationship between Net Public Foreign Assets and Growth Over Time Excluding Flows from China



(b) International Reserves and Public External Debt to GDP excluding Flows from China (simple average)



Notes: This figure plots the resulting coefficient β excluding flows from China from equation (4). The coefficient is the blue line, and the gray lines are a 90 percent confidence interval. *Source:* IDS World Bank, WDI World Bank, BOP IMF, Horn et al. (2021) and authors' calculations.

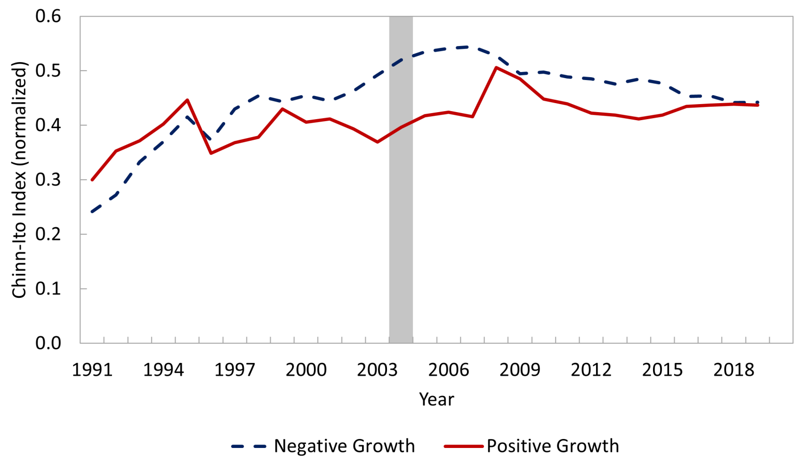
4.5 Flow of Funds from China

Upon the launch of the Belt and Road Initiative in 2013, China pledged to increase infrastructure investment in the developing world. Beginning in 2017, the Belt and Road Initiative investment projects were estimated to add over 1 trillion USD of outward funding for foreign infrastructure over a 10-year period (OECD (2018)).

Efforts have been made to make data on Chinese lending practices available, but data is scarce and the process of lending is not transparent. A string of recent papers (Horn et al., 2021; Horn et al., 2022; Guler et al., 2022; Horn et al., 2023) shed light on the lending practices that China has with the developing world. These authors show that the increase in debt from China to emerging market and low-income developing countries has started to reshape the international financial system. Given the massive scale of the Chinese flows and that those flows are directly to sovereigns, the Belt and Road initiative could potentially explain the reversal between growth and net public foreign assets.

Using the data from Horn et al. (2021) as well as the data reported to the IDS we calculate the amount of debt that China has lent to countries over the years. *Figure 14a* shows that excluding flows from China does not alter the observed change in the correlation between growth and net public foreign assets. Moreover, *Figure 14b* shows that the stark decrease in public external debt and the increase in international reserves remain even in the absence of Chinese lending.

Figure 15: Chinn-Ito Index (simple average)



Source: Chinn and Ito (2006) and authors' calculations.

4.6 Capital Mobility

The last potential explanation we consider is a mechanism analyzed in Gourinchas and Jeanne (2013). More financially open economies experiencing higher growth rates should attract more capital. That is, have larger net financial asset positions. However, Gourinchas and Jeanne (2013) found the opposite to be true, economies with higher growth rates and greater financial openness attracted less capital. One possible explanation for the reversal in the relationship between net public foreign assets position and growth is that economies with higher levels of financial openness are attracting more capital. Using the index constructed by Chinn and Ito (2006), *Figure 15* plots the evolution of financial openness over time. A higher index indicates fewer capital controls in place. After 2004 there is not a clear difference in the capital openness index between low and high growth countries. This suggests that changes in the movement of capital across countries of differing growth levels cannot be attributed to changes in capital controls during this period.

5 Quantitative Illustration

In this section we quantitatively decompose the role of an increased productivity variance plays in explaining the reversal of correlation between growth and public net foreign assets. To do so, we begin by calibrating a stochastic open-economy neoclassical growth model to match important features of the pre *and* post-2004 period. We then use this model to simulate the effects of an unanticipated increase in productivity variance for low growth countries.

5.1 Model

In this section we outline a simple theoretical framework used to illustrate the mechanism underlying the reversal of the allocation puzzle. We follow the neoclassical growth model with wedges laid out in Gourinchas and Jeanne (2013) with the sole addition of uncertainty over the productivity path.

The model describes a small open economy that can borrow and lend, given some exogenously determined gross real world interest rate R^*. Throughout, let upper case letters refer to aggregate variables, and lower-case letters refer to per capita variables. Time is discrete and indexed by t. In each period t, the economy experiences one event $s_t \in S$. Let s^t denote the history of events from 0 to t. The probability of history s^t at date t is given by $\pi(s^t)$. The economy produces a single final good using capital and labor according to,

$$Y_t(s^t) = z_t(s^t) A_0 K_t(s^{t-1})^{\alpha} (\gamma^t N_t)^{(1-\alpha)} \tag{5}$$

where $K_t(s^{t-1})$ is the stock of physical capital accumulated domestically, N_t is the deterministic labor supply, A_0 the level of initial productivity, γ^t the deterministic trend in labor augmenting productivity, and $z_t(s^t)$ a random variable with bounded support governing the stochastic component of productivity. Hence, there is uncertainty in the model with respect to the production of aggregate output $Y_t(s^t)$.

The country can accumulate non-state contingent foreign bonds or issue external debt. Hence, the aggregate resource constraint is given by,

$$C_t(s^t) + I_t(s^t) + R^* D_t(s^{t-1}) = Y_t(s^t) + D_{t+1}(s^t) \tag{6}$$

where $D_t(s^{t-1})$ is the stock of external debt, and $I_t(s^t)$ is investment. We are assuming that there is no default risk, and hence the country pays the risk-free rate on its stock of debt. Additionally, we make the simplifying assumption that the level of debt $D_t(s^{t-1})$ is unconstrained.[8] The law of motion for capital is given by,

$$K_{t+1}(s^t) = I_t(s^t) + (1-\delta) K_t(s^{t-1}) \tag{7}$$

with the depreciation of capital given by δ. In period t, changes in the net foreign asset position are given by domestic investment minus domestic savings,

$$D_{t+1}(s^t) - D_t(s^{t-1}) = I_t(s^t) - \big(Y_t(s^t) - (R^* - 1) D_t(s^{t-1}) - C_t(s^t)\big) \tag{8}$$

[8]See Gourinchas and Jeanne (2013) for a detailed discussion of this assumption.

The marginal product of capital less depreciation is given by,

$$R_t(s^t) = z_t(s^t)\alpha A_0\left(\frac{k_t(s^{t-1})}{\gamma^t}\right)^{\alpha-1} + 1 - \delta \tag{9}$$

Note that with the presence of uncertainty over the productivity process, the private return to domestic capital $R_t(s^{t-1})$ does not equal the world risk free rate.

We now turn to the discussion of domestic consumption and savings. The framework outlined here follows a classic Cass-Ramsey model extended to allow for uncertainty and population growth. Population N_t grows at the exogenous rate η, according to $N_t = N_0\eta^t$. The representative agent maximizes expected lifetime utility according to,

$$U_t = \sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi_t(s^t)N_t u(c_t(s^t)) \tag{10}$$

where the period utility is given by the CRRA function $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, with σ as the coefficient of risk aversion. The number of agents in the model is normalized to be of mass one, and hence per capita and aggregate variables coincide. As in Gourinchas and Jeanne (2013) the negative correlation can be generated by introducing "wedges" distorting the agent's first-order conditions. These wedges will be used to calibrate the model to observed data for the pre-2004 period as well as the 1980-2019 period.

We introduce two wedges, one distorting the capital accumulation decision and one the savings decision. The wedge on capital, τ_k acts as a tax on capital income, where investors do not receive the entire gross return to capital $R_t(s^t)$. Alternatively, it can also be thought of as some distortion such as credit market imperfections, corruption, or as in Aguiar and Amador (2011) the risk of expropriation. The savings wedge, τ_s will act as a tax on capital income and distorts the agent's intertemporal decision over debt and capital accumulation. The representative agent's budget constraint is given by,

$$\begin{aligned} &C_t(s^t) + K_{t+1}(s^t) = \\ &\quad (1-\tau_s)(R_t(s^t)(1-\tau_k)K_t(s^{t-1}) - R^*D_t(s^{t-1})) + D_{t+1}(s^t) + N_t(w_t(s^t) + T_t(s^t)) \end{aligned} \tag{11}$$

where, $T_t(s^t)$ is a lump-sum rebate of tax revenues to the household. To focus solely on the distortions introduced by the wedges, we assume the per capita lump-sum transfer is given by $T_t(s^t) = (\tau_s + \tau_k - \tau_s\tau_k)R_t(s^t)k_t(s^{t-1}) - \tau_s R^* D_t(s^{t-1})$. $w_t(s^t)$ is the wage, given by the marginal product of labor,

$$w_t(s^t) = (1-\alpha)z_t(s^t)A_0 k_t(s^{t-1})^{\alpha}(\gamma^t)^{1-\alpha} \tag{12}$$

The representative agent then maximizes expected lifetime utility (10) subject to the budget constraint (11). This yields the Euler equations for the small open economy,

$$c_t(s^t)^{-\sigma} = \beta(1-\tau_s)(1-\tau_k)\mathbb{E}[R_{t+1}(s^{t+1})c_{t+1}(s^{t+1})^{-\sigma}] \tag{13}$$

$$c_t(s^t)^{-\sigma} = \beta(1-\tau_s)R^*\mathbb{E}[c_{t+1}(s^{t+1})^{-\sigma}] \tag{14}$$

Let γ^* denote the constant rate at which the world productivity frontier grows. That is, we assume that the rest of the world is composed of advanced economies which have achieved a deterministic balanced growth path. If we then additionally assume that the rest of the world has the same preferences as the representative agent, the world interest rate is given by,

$$R^* = \frac{\gamma^{*\sigma}}{\beta} \tag{15}$$

Note, the household decision, and predicted level of net foreign assets $D_t(s^{t-1})$ and investment rate $I_t(s^t)$ are a function of τ_s and τ_k.

In this economy, a country is characterized by an initial per capita capital stock k_0, per capita debt d_0, population growth rate η, a deterministic productivity path $A_0\gamma^t$, and a stochastic productivity path $\{z_t(s^t)\}_{t=0}^{\infty}$. It is assumed that all countries are financially open at time $t = 0$. The model is then used to estimate the level and direction of international financial asset flows from $t = 0$ onward.

The distorted model presented above is used to generate negative correlation between net foreign assets and growth. The sole difference in the undistorted model is in the budget constraint of the representative household. In this case, the household decision and hence model's predicted level of net foreign assets $D_t(s^{t-1})$ and investment rate $I_t(s^t)$ are not a function of τ_s and τ_k. In the undistorted model the correlation between net foreign assets and growth is positive.

5.1.1 Incomplete Asset Markets

In an incomplete market setting, risk cannot be fully insured across countries, even with access to international credit markets. Under uncertainty, the country faces precautionary savings motives to insure against future income shocks. Given that it cannot perfectly insure itself against negative TFP shocks, an increase in the standard deviation of the stochastic productivity component $z_t(s^t)$ generates an increase in the variance of consumption. This leads to higher savings (or reserves accumulation) for protection against higher volatility.

If asset markets were complete, where the government can buy state contingent Arrow-Debreu securities, a higher standard deviation in the productivity component would imply that even under the realization of a negative TFP shock, capital would flow to the country. This results from perfect consumption risk sharing. This means that under complete markets the correlation between growth and net foreign assets would become more negative, unlike the trend reversal seen in the post-2004 period, when an increase in TFP volatility is observed.

A number of papers (Backus and Smith, 1993; Kollmann, 1995; Ravn, 2001; Fitzgerald, 2012) have studied the failure of perfect international consumption risk sharing across countries. Explanations for the failure of risk sharing include: the costs of trading goods internationally or deviations in international asset markets from the Arrow-Debreu benchmark (i.e. complete markets). In particular, Fitzgerald (2012) finds that frictions in international asset markets significantly impede optimal consumption risk sharing between rich and poor countries. Hence, we believe that the assumption of incomplete markets is empirically realistic. Coeurdacier et al. (2020) analyze the welfare gains from financial integration and risk sharing in a stochastic neoclassical growth model, and show that financial integration can generate a reversal in capital flows.

5.2 Calibration

Unlike the deterministic model of Gourinchas and Jeanne (2013), the version here with uncertainty does not permit an analytical solution to directly solve for wedges which generate the observed data. We do not calibrate wedges for each of the 64 countries in our sample individually.[9] Instead, we divide our sample into five groups of countries according to productivity growth over the 1980 to 2004 period and the 2005 to 2019 period. This defines five countries by differing levels of growth to which we calibrate the model for two different time periods. Details are described below in *Sections 5.2.1* and *5.2.2*.

To illustrate our mechanism, we argue that countries faced an unanticipated shock to their steady state, or an "MIT shock". This can be interpreted as an exogenous change that countries did not foresee. There is a large macroeconomic literature that has used MIT shocks to account for exogenous events (Abel and Blanchard, 1983; Boppart et al., 2018; Boar and Midrigan, 2022). An underlying assumption is that after the unexpected increase in volatility, the country does not expect another shock to occur. This exercise compares two steady states, the pre-2004 and post-2004 steady states. In future work we believe it would be interesting to analyze the entire transition path.

The main quantitative exercise involves picking parameters in the model for each group of countries to replicate net public capital flows, investment rates, and output volatility in the data. We calibrate the model to match the pre-2004 and post-2004 periods. The calibrated

[9] This is done for two reasons. First, to avoid imposing unrealistically strong assumptions on the stochastic productivity process. Second, for tractability purposes.

parameters for the post-2004 period are used for the counterfactual exercise. Calibrating the wedges introduced in *Section 5.1*, the model produces the negative correlation between growth and net public foreign assets observed in the data for the 1980-2004 period. A counterfactual economy is then computed in which we assume countries experienced an unexpected change in their TFP volatility, growth rates, and wedges. A decomposition is calculated to estimate the fraction of the change in correlation between net public foreign assets and growth that can be attributed to the change in TFP volatility.

5.2.1 External Calibration

Calibration of the exogenously set parameters closely follows Gourinchas and Jeanne (2013) and the developmental accounting literature (e.g., Caselli, 2005). We first describe the externally set parameters that are constant across our groups of countries. The depreciation rate δ is set to 0.06, and the capital share of income α is set to 0.33. We assume that the world productivity frontier is given by U.S. total factor productivity growth over the 1980 to 2004 period and set $\gamma^* = 1.017$. Assuming $\sigma = 1$ (i.e. log preferences) and a time discount factor of $\beta = 0.96$, the world interest rate is $R^* = 1.059$.[10] We ignore population growth across countries, as it is not essential to our question of interest. As in Gourinchas and Jeanne (2013) the model predictions will be computed assuming that there is no initial debt and initial capital stock is given by the steady state level (of the deterministic model).

It is assumed that the random variable $z(s^t)$ which determines the stochastic component of the productivity path is governed by the first-order autoregressive (AR(1)) process,

$$\log(z_{t+1}) = \rho \log(z_t) + \epsilon_t \tag{16}$$

where ρ determines the persistency of aggregate output shocks, and $\epsilon_t \sim N(0, \sigma_z^2)$. The variance of $z(s^t)$ is controlled by σ_z^2. To estimate ρ we run the following regression for all countries in our sample,

$$\log(y_t) = \kappa \log(y_{t-1}) + \xi_t \tag{17}$$

where y_t is detrended GDP per capita. The estimated parameter κ then gives a $\rho = 0.87$. That is, we assume $\rho = \kappa$. The variance parameter σ_z^2 is internally calibrated so that the model matches output volatility in the data. This will be discussed further in *Section 5.2.2*.

The deterministic component of labor augmenting productivity growth for our five groups of countries is reported in *Table 1* for both time periods. These growth rates are obtained

[10] This set of parameters are constant for both time periods.

Table 1: Average productivity growth rates for country groups

Country Group	Growth Rate 1980-2004	Growth Rate 1980-2019
γ_1	0.962	0.989
γ_2	0.987	1.001
γ_3	0.999	1.012
γ_4	1.015	1.022
γ_5	1.042	1.041

by first ranking all countries by average productivity growth from 1980-2004 and from 2005-2019, and then dividing them into five equal groups.[11] Productivity growth rates are then averaged across countries within each group.

5.2.2 Internal Calibration

There are 15 parameters remaining in the model to calibrate. A vector of savings wedges $\tau_s = [\tau_{s,1}, \tau_{s,2}, \tau_{s,3}, \tau_{s,4}, \tau_{s,5}]$, capital wedges $\tau_k = [\tau_{k,1}, \tau_{k,2}, \tau_{k,3}, \tau_{k,4}, \tau_{k,5}]$, and variances of the AR(1) process $\sigma_z^2 = [\sigma_{z,1}^2, \sigma_{z,2}^2, \sigma_{z,3}^2, \sigma_{z,4}^2, \sigma_{z,5}^2]$ for each of the five country groups and each of our *two* time periods. The parameters are chosen to minimize the distance between 15 moments simulated by the model and empirical counterparts observed in the data during the 1980-2004 period and the 2005-2019 period.

The capital and savings wedges are identified as in Gourinchas and Jeanne (2013). The savings wedges control net capital inflows ${}^{\Delta D}/_{Y_0}$. These five moments together with the five productivity growth rates pin down the correlation between net public foreign assets and growth. The capital wedges are used to target five moments of average investment to output ratios (i_k).

Using gross fixed capital formation data from the WDI we calculate average investment as average investment to GDP for the periods 1980 to 2004 and 2005 to 2019 for each country. Using IDS data for the public external debt, we follow Gourinchas and Jeanne (2013) and calculate the change in the net public financial asset position as the change in the ratio of external debt to initial GDP. To mitigate the possibility that the data point is subject to year specific movements, we use the 5-year average of external debt from 2000 to 2004 and 2015 to 2019 as the endpoints, and the initial points as the 5-year average of external debt from 1980 to 1985 and 2000 to 2004. Then, using the externally calibrated productivity growth rates, countries are binned into their respective groups and simple averages are taken

[11] The productivity growth data is computed using the replication package from Gourinchas and Jeanne (2013), see *Appendix B.2* for a description of the data.

within each group. These averages are the targeted moments in the model.

Finally, the variances of the AR(1) process σ_z^2 are calibrated so that the model replicates the standard deviation of per capita output seen in the data for each set of country groups. This then implies an average standard deviation across all country groups.

Prior to using the model to illustrate our mechanism explaining the changing correlation between net public foreign assets and growth we verify that the model replicates key moments across countries for both the 1980-2004 period and the 2005-2019 period. *Table 2* reports the calibrated values for these parameters, and *Table 3* reports the model fit compared to the data. The model can produce accurate targeted moments. The changes in net foreign assets simulated by the model imply a correlation between growth and changes in net public foreign assets of -0.53 compared to -0.52 in the data for the period 1980-2004.[12]

We observe a strong negative correlation between the savings wedges and growth rates. This is in line with the findings of Gourinchas and Jeanne (2013).[13] For the model to accurately generate the observed changes in capital inflows, the savings of high growth countries must be subsidized and low growth countries taxed. For the capital wedges we see that the relationship is roughly flat across country growth levels.

As a robustness check, Appendix B employs an alternative strategy to calibrate our capital and savings wedges. We use the closed form solutions derived in Gourinchas and Jeanne (2013) to directly estimate τ_s and τ_k from the data. While not a quantitative example, we obtain similar results and the qualitative mechanism holds. Moreover, with these wedges estimated at the individual country level we show that the relationship between growth rates and distortions is the same as generated by our model with country groupings.[14]

5.3 Decomposition

In this section we use the theoretical model calibrated to match key moments of the 1980-2004 period, to investigate our proposed mechanism explaining the reversal in correlation between net public foreign assets and growth.

The mechanism is simple. Beginning in 2004, low growth countries experienced increased productivity risk as seen in *Figure 8*. We argue that this caused a flight to safer assets. In

[12]The correlations in the data are the correlation from the five country groupings, but these numbers are quite similar to the correlations reported in *Figure 1*. For the period 1980-2004 the correlation is -0.55.

[13]See *Appendix B* for a discussion on how Gourinchas and Jeanne (2013) estimate the wedges and how our findings compare to theirs.

[14]See Appendix D for a description of the computational algorithm.

Table 2: Internally calibrated parameters

Parameter	1980-2004 Value	2005-2019 Value
$\tau_{s,1}$	0.0589	0.0357
$\tau_{s,2}$	0.0344	0.026
$\tau_{s,3}$	0.0209	0.0091
$\tau_{s,4}$	0.0075	0.0005
$\tau_{s,5}$	-0.02131	-0.0203
$\tau_{k,1}$	0.0478	0.0615
$\tau_{k,2}$	0.065	0.031
$\tau_{k,3}$	0.055	0.04
$\tau_{k,4}$	0.054	0.05
$\tau_{k,5}$	0.062	0.046
$\sigma^2_{z,1}$	0.022	0.0372
$\sigma^2_{z,2}$	0.015	0.03
$\sigma^2_{z,3}$	0.017	0.02
$\sigma^2_{z,4}$	0.02	0.024
$\sigma^2_{z,5}$	0.02	0.0175

Notes: This table reports internally calibrated values for savings wedges τ_s, capital wedges τ_k, and variance of productivity shocks σ^2_z. Wedges are reported from lowest growth country group (1) to highest growth country group (5).

Table 3: Model fit for targeted moments

		1980-2004		2005-2019	
	Moment	Model	Data	Model	Data
Δ Net Public Capital Flows	$\left(\frac{\Delta D}{Y_0}\right)_1$	0.396	0.391	0.032	0.030
	$\left(\frac{\Delta D}{Y_0}\right)_2$	0.495	0.492	0.034	0.036
	$\left(\frac{\Delta D}{Y_0}\right)_3$	0.141	0.151	0.201	0.195
	$\left(\frac{\Delta D}{Y_0}\right)_4$	0.482	0.484	-0.049	-0.049
	$\left(\frac{\Delta D}{Y_0}\right)_5$	0.074	0.097	0.032	0.033
Average Investment Rate	$i_{k,1}$	0.235	0.238	0.206	0.202
	$i_{k,2}$	0.197	0.200	0.251	0.255
	$i_{k,3}$	0.215	0.215	0.241	0.239
	$i_{k,4}$	0.210	0.213	0.216	0.213
	$i_{k,5}$	0.194	0.194	0.223	0.222
Output Standard Deviation	$SD_1(y)$	0.122	0.135	0.185	0.211
	$SD_2(y)$	0.078	0.076	0.169	0.170
	$SD_3(y)$	0.092	0.094	0.112	0.116
	$SD_4(y)$	0.107	0.108	0.129	0.130
	$SD_5(y)$	0.105	0.106	0.096	0.092

Notes: This table reports the model fit for changes in net capital inflows $\frac{\Delta D}{Y_0}$, average investment rates i_k and the standard deviation of per capital output $SD(y)$. Moments are reported from lowest growth country group (1) to highest growth country group (5).

Table 4: Average Productivity volatility by country groupings and time period

Country Group	1980-2004	2005-2019
σ_1	0.135	0.211
σ_2	0.076	0.170
σ_3	0.094	0.116
σ_4	0.108	0.130
σ_5	0.106	0.092

Notes: This table reports the average standard deviation of productivity across country groupings. TFP is demeaned and a 5-year backward rolling average is taken to obtain the standard deviation of each country. Moments are reported from lowest growth country group (1) to highest growth country group (5).

the data this is seen as a substitution from riskier debt to safer international reserves.[15] In the model, this would be seen as a substitution from borrowing towards (precautionary) savings. That is, the net public foreign asset position of a country should decrease.

We begin with the model calibrated to the 1980-2004 period, which generates a negative correlation of -0.53 between net public foreign assets and growth, the same as observed in the data. *Table 4* shows the standard deviation of productivity for our five country groups for the two time periods of interest. This table shows that variances decreased slightly for the highest growth country (γ_5), increased somewhat for the mid-growth countries (γ_4 & γ_3), and increased dramatically for the two lowest growth countries (γ_2 & γ_1) during the post-2004 period.

For the counterfactual exercise, we decompose the changing correlation into the contribution from increased TFP volatility and the residual (i.e. the growth rates and wedges). The starting point is the average correlation between net public foreign assets and growth between 1999-2004 as seen in the data (-0.55). The end point is the average correlation between net public foreign assets and growth for the period 2015-2019 from the data (-0.07).

Given that the model is non-linear, we must determine the contribution of changing productivity variance in two stages. In both stages, we begin with the calibrated model from 1980-2004 which replicates the correlation for that period. Starting from this model we include the calibrated TFP volatility, σ_z^2, from 2005-2019 and re-solve the model, holding all other parameters fixed at the 1980-2004 level and calculate the resulting correlation.[16]

[15] Public external debt is subject to risk of default and is riskier than international reserves. Note that in the simple model we are abstracting from a separate decision between international reserves accumulation and debt accumulation. In the model we only consider the overall net public foreign asset position. See Bianchi et al. (2018) for a richer model of international reserves and public external debt.

[16] The model is solved *without* linearizing, and the mean of the AR(1) process is kept fixed. This ensures that the changing correlation is attributable solely to the changing variance of the AR(1) process.

Following this, we introduce the growth rates and wedges calibrated for the period 2005-2019, recalculating the correlation. For the second stage, we change the order of introducing volatility and residuals. Initially, growth rates and wedges to the calibrated values for the period 2005-2019 are introduced to the 1980-2004 calibrated model, while keeping the standard deviation of the TFP process fixed at the 1980-2004 values and the correlation is calculated. Then we include the volatility parameter and compute the final correlation. The contribution of volatility is then determined as the average of these two exercises.[17]

These two exercises allow us to calculate how much the change in volatility contributed to the reversal of the correlation between net public foreign assets and growth. When the agents face higher uncertainty, keeping the other parameters fixed, they borrow less and save more. In the model this translates into a lower net foreign asset position and an increased correlation between growth and public net foreign assets. We find that the increase in productivity volatility accounts for approximately 46% of the change in correlation observed in the data. This counterfactual exercise quantitatively shows that the simple mechanism proposed in *Section 3* can explain nearly half the change in correlation between net public foreign assets and growth observed in the period 1980-2019.

6 Conclusion

This paper documents a novel and puzzling stylized fact and provides strong empirical and quantitative evidence as an explanation. The correlation between net public foreign assets and growth has dramatically changed over time. Beginning in 2004, the correlation changed from negative to zero or even weakly positive.

Mechanically, the changing correlation was caused by a sharp substitution away from debt, towards international reserves, for low growth countries. We show that low growth countries simultaneously experienced a pronounced increase in aggregate risk. Hence, we suggest a simple mechanism: precautionary savings.

Empirically, we lend evidence to this mechanism by ruling out several other possible explanations: (1) heavily indebted poor countries initiative, (2) commodity exporters, (3) central bank independence, (4) market exclusion, (5) flows of funds from China, and (6) capital mobility. Quantitatively we demonstrate this mechanism using a stochastic, open-economy neoclassical growth model. Using wedges on capital and savings, we calibrate the model to match the negative correlation observed in the data during the pre-2004 period.

[17] As a robustness exercise we calculated the contribution of each set of parameters to the change in correlation. We calculated the full six permutations when changing the TFP volatility, the growth rates, and the savings and capital wedges. When splitting the residual into the two components, the contribution from the TFP volatility remains largely unchanged.

We then consider a counterfactual economy with higher levels of uncertainty for low growth countries and calibrated wedges to the 2005-2019 period. This exercise shows that an empirically accurate increase in productivity volatility accounts for 46% of the observed changes in the correlation between growth and net public foreign assets.

The analysis in this article uncovered several interesting findings that were beyond the scope of this paper but are promising avenues for future work. While this paper focused on the flow of public funds, there was also a reversal in correlation between private capital flows and growth. This correlation in the post-2004 period is no longer positive. We also find that the relationship between net foreign assets and growth has been reversed in the post-2004 period leaving the question open of whether the reversal is coming from private or public flows.

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Online Appendix

Appendix A Data

This section discusses details of each dataset used, and the variables used in each of these datasets. All data is publicly available.

A.1 World Development Indicators

The World Development Indicators (WDI) is the primary World Bank collection of development indicators, compiled from officially recognized international sources. It presents current data available, and includes national, regional, and global estimates.

The variables we take from the WDI are GDP in current US dollars, constant GDP per capita, and GDP per capita in US dollars.

A.2 Penn World Table

The Penn World Table is a database with information on relative levels of income, output, input and productivity, covering 183 countries between 1950 and 2019.

The variables we use from the Penn World Table are capital stock at constant prices, the share of labor compensation in GDP, population, and output-side real GDP at chained PPPs.

A.3 International Debt Statistics

The International Debt Statistics (IDS) is a dataset by the World Bank that reports the long-term external debt owed by a public agency or a private agency with a public guarantee. The IDS reports aggregates on long-term external debt owed by the private sector with no public guarantee for 121 low- and middle-income countries. The countries report to the World Bank Debt Reporting System (DRS).

From the IDS we use the total public and publicly guaranteed debt, the public and publicly guaranteed debt by official creditors, the public and publicly guaranteed debt by private sector, and the public and publicly guaranteed debt owed to China (bilateral).

A.4 World Development Outlook

The World Economic Outlook (WEO) database from the International Monetary Fund (IMF) contains selected macroeconomic times series from the statistical appendix of the World Economic Outlook (WEO) report. The WEO report presents the IMF's analysis and projections of economic developments at the global level, for major country groups and many individual countries. The WEO is released in April and September/October each year.

From the WEO we use the country classification for commodity exporters as well as countries that participated in the HIPC initiative.

A.5 International Financial Statistics

The International Financial Statistics (IFS) data from the IMF is the principal statistical dataset from the IMF. The available indicators are a country's exchange rate, international liquidity, monetary statistics, interest rates, prices, production, government accounts, national accounts, among others.

From the IFS we take international reserves minus gold.

A.6 Balance of Payments

The Balance of Payments and International Investment Position Statistics (BOP/IIP) data from the IMF reports transactions between residents and non-residents during a given year. The data consists of the goods and services account, the primary income account, the secondary income account, the capital account, and the financial account.

From the BOP we take the current account.

A.7 Annual Report on Exchange Arrangements and Exchange Restrictions

The Annual Report on Exchange Arrangements and Exchange Restrictions (AREAER) tracks the exchange rate and trade regimes of all members of the IMF and the following territories: Aruba, Curaçao and Sint Maarten and Hong Kong SAR and Macao SAR for a total of 194 economies.

We use the variables which report exchange rate regime.

A.8 Central Bank Independence Index

The Central Bank Independence Index developed in Garriga (2016) is a dataset on de jure central bank independence (CBI), which includes yearly data for 182 countries between 1970 and 2020. The dataset identifies statutory reforms affecting CBI, their direction, and the attributes necessary to build the Cukierman, Webb and Neyapty index.

We use the weighted central bank independence index for our analysis.

A.9 Defaults with the Private Sector

The dataset compiled in Asonuma and Trebesch (2016) has information on the duration of defaults and the restructuring processes between governments and their foreign private creditors (external banks and bondholders) at a monthly frequency. The dataset begins in June 1975 and ends in September 2020. It covers all restructurings until mid-2020, a total of 196 events, plus ongoing cases.

We take the length of default episodes in the dataset by country to compute statistics on default episodes with the private sector.

A.10 Capital Openness Index

The Chinn-Ito index is an index measuring a country's degree of capital account openness. The index was initially introduced in Chinn and Ito (2006). The indexed is constructed by observing the restrictions on cross-border financial transactions reported in the IMF's Annual Report on Exchange Arrangements and Exchange Restrictions (AREAER).

From these data we take the capital account index to show how capital controls across the world have evolved in time.

A.11 External Wealth of Nations Mark II Database

The External Wealth of Nations Mark II database reports estimates of external assets and liabilities for 211 countries for the period 1970-2022. The data includes net and gross external positions, composition of international portfolios, distinguishing between foreign direct investment, portfolio equity investment, official reserves, and external debt.

Lane and Milesi-Ferretti (2007) show that these numbers are more accurate than those reported in the IMF BOP dataset given the large differences in the errors and omissions category.

A.12 China Debt Stock Database

The China Debt Stock Database contains estimates of debt stocks owed to China for 107 countries from 2000 to 2017. The estimates of debt owed to China are based on loan-level data compiled from various sources. The authors merge this data on loans with creditor and debtor-specific information on interest rates, grace periods and maturities to construct loan-specific repayment schedules. By aggregating loan-specific cash flow streams, they can estimate interest and amortization payments and thus the outstanding debt stock on a country-year basis.

We take the total stock of debt owed to China and compare it to the data reported to the World Bank to construct a total measure of debt owed to China. Finally, we subtract the stocks of debt owed to China from the overall public external debt for the measure of flows without China.

Appendix B Robustness

In this section we present robustness checks in addition to those presented in *Section 4*. *Appendix C* reports further robustness analysis.

B.1 Alternate Estimation of Wedges

Updating the data used in Gourinchas and Jeanne (2013) allows us to estimate the savings and capital wedges following identical methodology. We show that using these estimated wedges leads to the same conclusion as in our main quantitative illustration with the calibrated wedges. *Section 4* of Gourinchas and Jeanne (2013) describes the model with wedges that generates a negative correlation between net foreign asset and growth. This correlation is generated through the introduction of "wedges" that distort the agent's first-order conditions. See *Section 5.1* for details.

The household decision and hence model's predicted level of net foreign assets D_t and investment rate I_t are a function of τ_s and τ_k. *Proposition 2* in Gourinchas and Jeanne (2013) gives a closed form solution that can be used to solve for the capital wedge τ_k.

Figure B.1 shows the estimated capital wedges τ_k which range from -9.0% (Nigeria) to 45.4% (Egypt) with an average of 11.7%. The estimated capital wedges are close to the wedges originally estimated by Gourinchas and Jeanne (2013). *Figure B.1* plots the relationship between the capital wedges and productivity catch-up.[18] We find that capital wedges (with the updated data) display no relationship with productivity catch-up. This further validates our findings when we estimated capital wedges from the quantitative model.

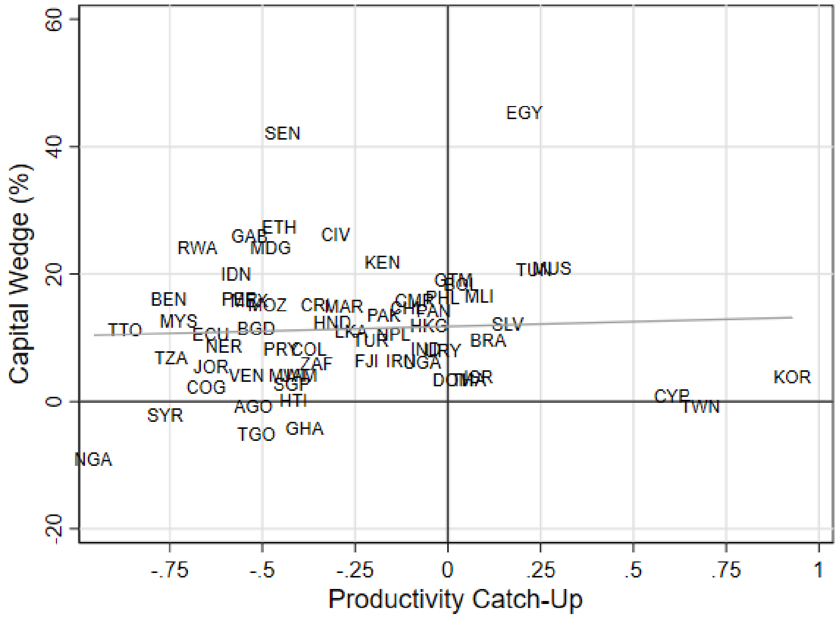
Section 1 in the Online Appendix of Gourinchas and Jeanne (2013) describes the closed form solution used to solve for the savings wedges τ_s. The estimated saving wedges τ_s are plotted in *Figure B.2*. The savings wedges range from -7.1% (Botswana) to 14.8% (Nigeria) with an average wedge of 2.3%. As in the original paper, the saving wedge is negatively correlated with productivity catch-up. Again, this validates the results of the internal calibration used in our quantitative model and counterfactual exercise.

B.2 Allocation Puzzle

Using the replication package provided by Gourinchas and Jeanne (2013) we update their data to the data used in this paper. We verify that we can use our updated data to replicate

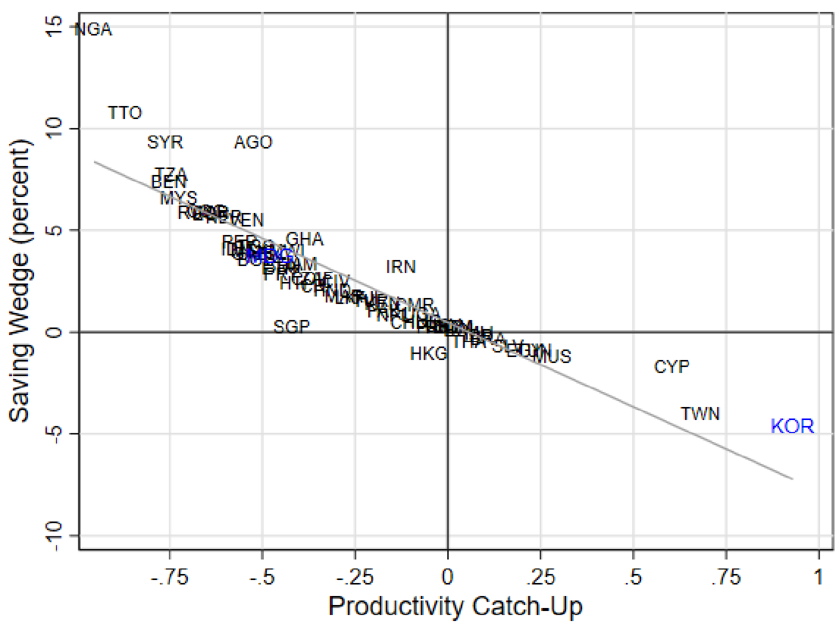
[18] Productivity catch-up is defined in Gourinchas and Jeanne (2013) to capture the gap between domestic productivity and U.S. productivity. That is, productivity catch up for country j at time t is given by, $\pi_{j,t} \equiv \frac{A_{j,t}}{A_{us,t}} - 1$. As previously discussed, Alfaro et al. (2014) show that our analysis using growth rates is equivalent to that of Gourinchas and Jeanne (2013) using productivity catch-ups.

Figure B.1: Productivity Catch-up and Capital Wedge



Source: Authors' calculations based on Gourinchas and Jeanne (2013).

Figure B.2: Productivity Catch-up and Savings Wedge



Source: Authors' calculations based on Gourinchas and Jeanne (2013).

the figures of Gourinchas and Jeanne (2013). This section discusses the data used as well as the replicated figures. All data used in Gourinchas and Jeanne (2013) is publicly available. The only variables that are no longer available are the variables from the Penn World Table that have been discontinued after the version 8 update. Gourinchas and Jeanne (2013) used version 6.1. To replicate their figures, we use the Penn World Table version 10.0. For the purpose of replicating their figures we make use of the nearest variables now reported in the Penn World Tables. *Figure B.3* replicates *Figure 1* in Gourinchas and Jeanne (2013) and finds a negative correlation between NFA and productivity growth, as in the original paper.

Figure 7 in Gourinchas and Jeanne (2013) breaks down the components of the net foreign asset position between public and private flows. We replicate their figures and show that with the updated data we find the same correlations. *Figure B.4* shows that for the year 2000 there is a negative correlation between net public foreign asset position and productivity catch up. There is a positive relationship between private net foreign asset position and productivity catch up (*Figure B.5*). Finally, breaking net public foreign asset position between public external debt and international reserves shows that both components are negatively correlated with growth, but the coefficient associated with international reserves is much larger, suggesting that the correlation is mainly driven by international reserves.[19]

One difference between the sample in Gourinchas and Jeanne (2013) and ours, is that the former includes some advance economies in their analysis of net public foreign assets (i.e., Korea, Israel, Singapore). The IDS does not report data on public external debt for advanced economies. The replication package shows that for these countries the authors replace the stock of public external debt to be zero. Note that in *Figure B.6* these countries have a change in their public and publicly guaranteed external debt of zero.

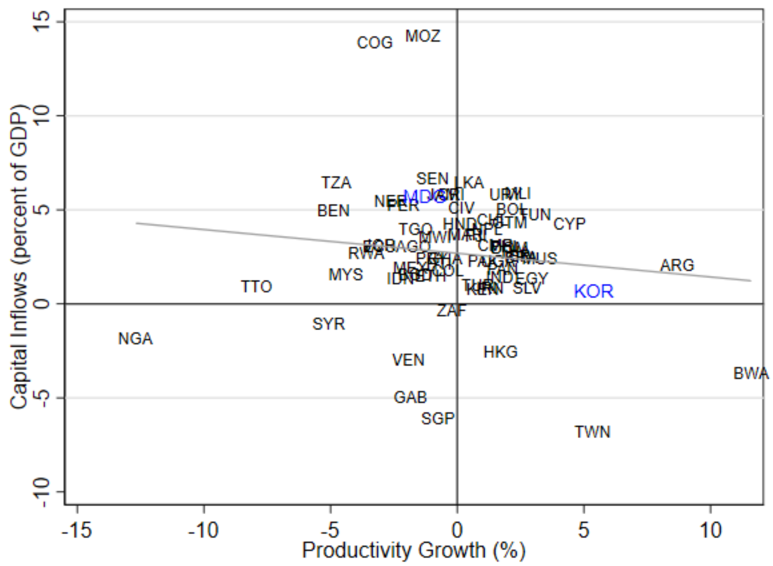
B.3 Sample of Countries

The sample of countries used in our analysis differs from the sample in Gourinchas and Jeanne (2013). *Table B.1* lists every country that is not in our sample and the reason why it was excluded from our analysis. As a robustness check we dropped all countries from our sample that are not in Gourinchas and Jeanne (2013) and all results remain unchanged.

Including China in the sample by assuming the level of public external debt in 1980 is the level observed in 1981, does not alter any results. Hence, we also conclude, China cannot explain changing trends in the correlation between growth and net public foreign assets.

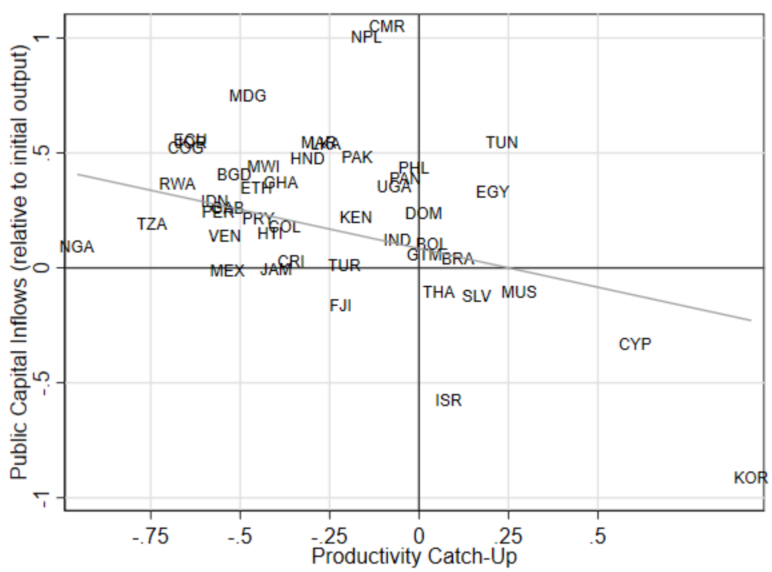
[19] As Gourinchas and Jeanne (2013) we remove Botswana and Singapore from the plots as they are outliers, including them in the figures only strengthens the relationship. In the updated data Argentina is also an outlier, so we remove it from the plots in the breakdown. As with Botswana and Singapore including them in the plots only strengthens the relationship.

Figure B.3: Average Productivity Growth and Average Capital Inflows between 1980 and 2000



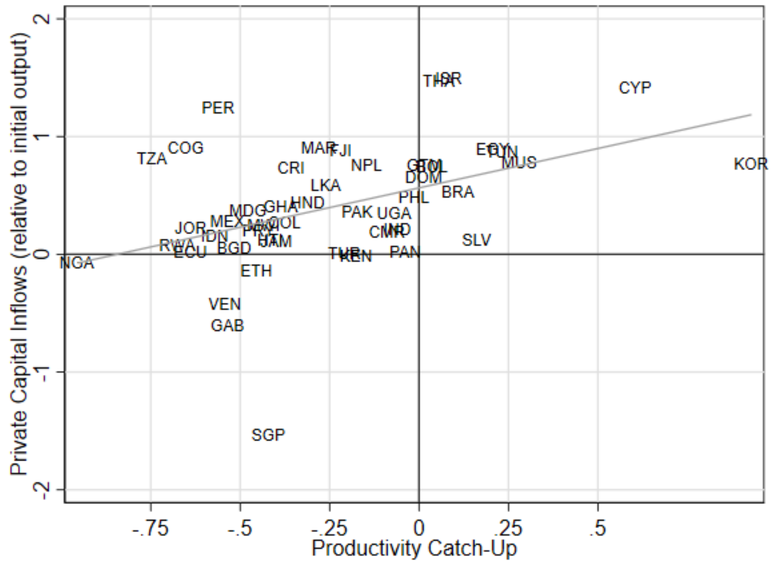
Source: Authors' calculations based on Gourinchas and Jeanne (2013).

Figure B.4: Productivity Catch-up and Change in Public External Debt



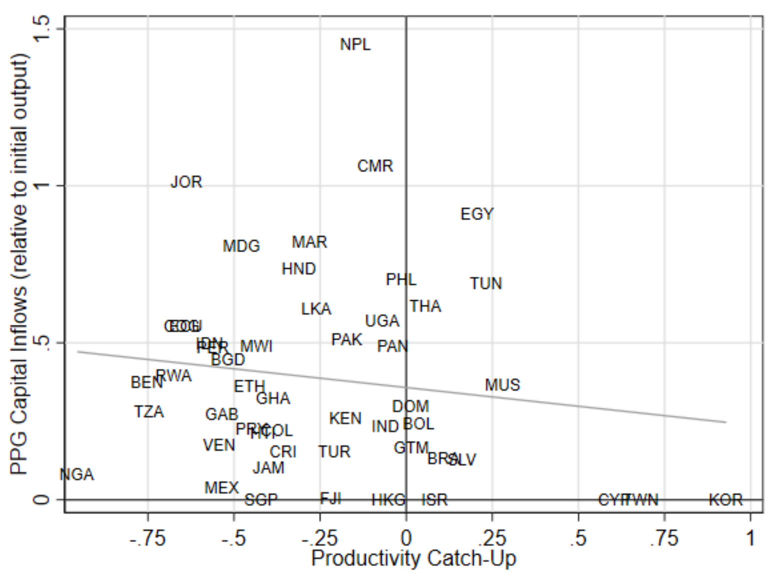
Source: Authors' calculations based on Gourinchas and Jeanne (2013).

Figure B.5: Productivity Catch-up and Change in Private External Debt



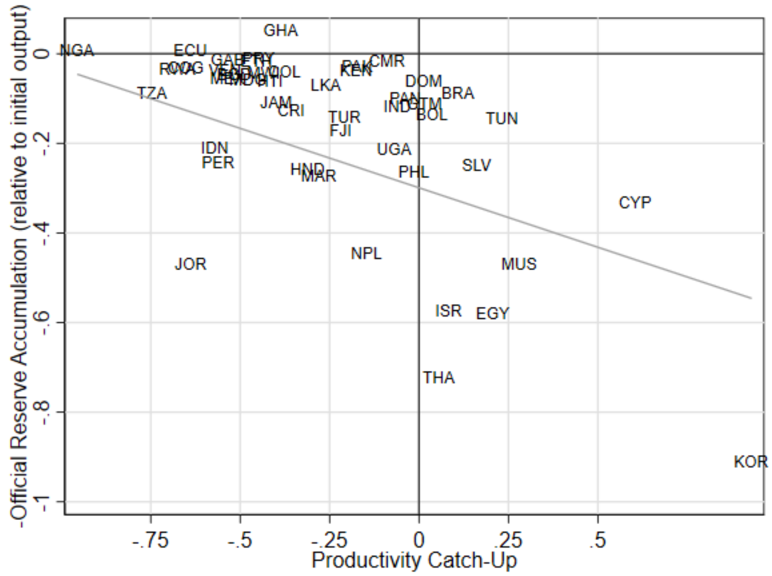
Source: Authors' calculations based on Gourinchas and Jeanne (2013).

Figure B.6: Productivity Catch-up and Change in Public and Publicly Guaranteed External Debt



Source: Authors' calculations based on Gourinchas and Jeanne (2013).

Figure B.7: Productivity Catch-up and Change in International Reserves



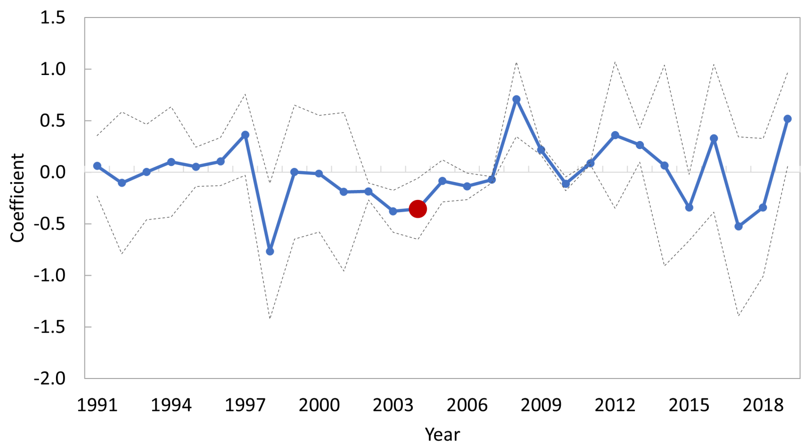
Source: Authors' calculations based on Gourinchas and Jeanne (2013).

Table B.1: Countries in Gourinchas and Jeanne (2013) sample that are absent in our sample

Country	**Reason for exclusion**
Angola	External debt data begins in 1989 and reserves data begins in 1995
Benin	Missing international reserve data
Chile	Missing external debt data
China	External debt data begins in 1981
Ivory Coast	Missing international reserve data
Iran	Missing international reserves data after 1982
Mali	Missing international reserves data
Mozambique	Missing External debt and international reserves data after 1984
Niger	Missing international reserves data
Senegal	Missing international reserves data
Syria	Missing external debt before 2008 and international reserves data after 2010
Togo	Missing international reserves data
Trinidad & Tobago	Missing external debt data
Uruguay	Missing external debt data
Venezuela	Missing GDP data after 2015 and international reserves data after 2017
South Africa	Missing external debt data before 1994
South Korea	Missing external debt data
Hong Kong	Missing external debt data
Taiwan	Missing external debt data
Singapore	Missing external debt data
Cyprus	Missing external debt data

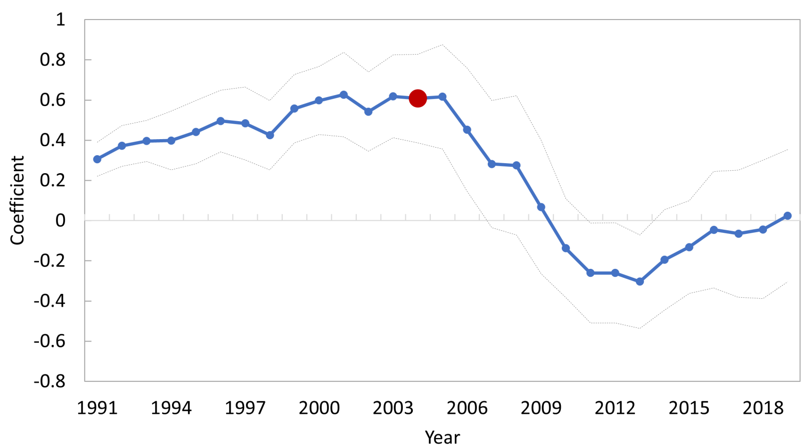
Appendix C Additional Figures and Tables

Figure C.1: Relationship between Net Foreign Assets and Growth Over Time



Source: Lane and Milesi-Ferretti (2018) and https://www.brookings.edu/articles/the-external-wealth-of-nations-database/. *Notes:* This figure plots the resulting coefficient β from equation (4) when the independent variable is the net foreign asset position. The coefficient is the blue line, and the gray lines are a 90 percent confidence interval.

Figure C.2: Relationship between Net Private Foreign Assets and Growth Over Time

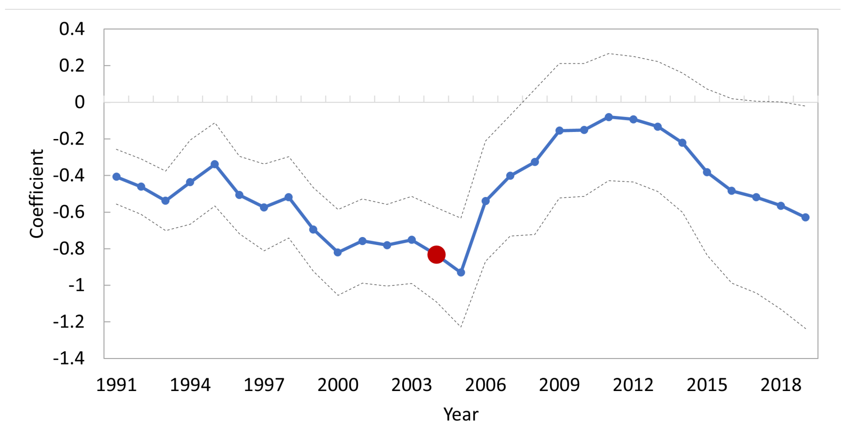


Notes: This figure plots the resulting coefficient β_T from equation (4) when the independent variable is the private flows from the current account. The coefficient is the blue line, and the gray lines are a 90 percent confidence interval.

Table C.1: Sample of countries used in regression analysis

Country	Country	Country	Country
Algeria	Egypt	Jordan	Paraguay
Argentina	El Salvador	Kenya	Peru
Bangladesh	Eswatini	Lebanon	Philippines
Belize	Ethiopia	Lesotho	Republic of Congo
Bolivia	Fiji	Madagascar	Rwanda
Botswana	Gabon	Malawi	Samoa
Brazil	The Gambia	Maldives	Sierra Leone
Burundi	Ghana	Mauritius	Solomon Islands
Cameroon	Grenada	Mexico	Sri Lanka
Central African Republic	Guatemala	Morocco	St. Vincent and the Grenadines
Colombia	Guyana	Nepal	Tanzania
Comoros	Haiti	Nicaragua	Thailand
Democratic Republic of Congo	Honduras	Nigeria	Tunisia
Costa Rica	India	Pakistan	Turkey
Dominican Republic	Indonesia	Chad	Uganda
Ecuador	Jamaica	Papua New Guinea	Zambia

Figure C.3: Relationship between Net Public Foreign Assets and Growth Over Time, base year 1970

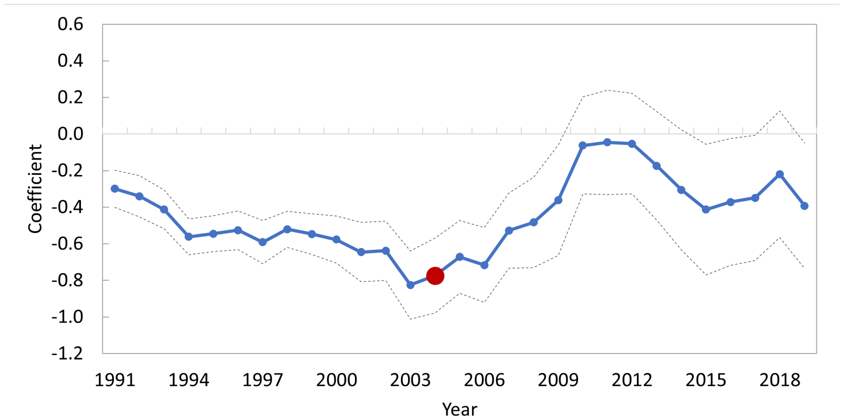


Notes: This figure plots the resulting coefficient β_T from equation (4) when the base year is 1970. The coefficient is the blue line, and the gray lines are a 90 percent confidence interval.

Table C.2: Sample of countries that participated in the HIPC initiative

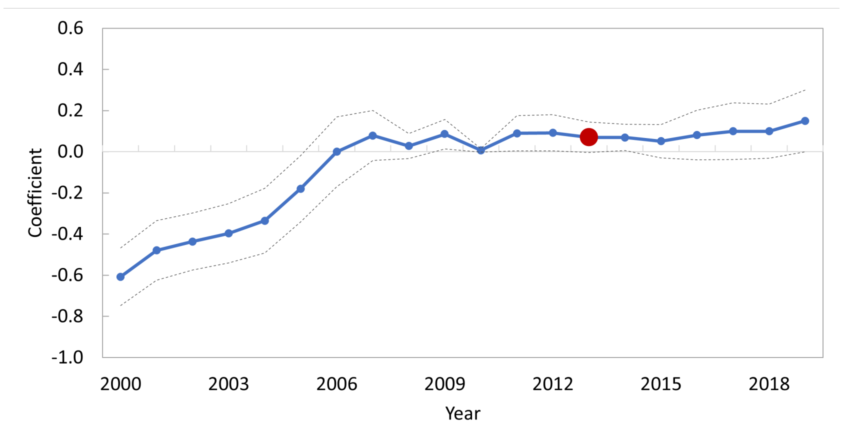
Country	Country
Bolivia	Haiti
Burundi	Honduras
Cameroon	Madagascar
Central African Republic	Malawi
Comoros	Nicaragua
Republic of Congo	Rwanda
Ethiopia	Sierra Leone
The Gambia	Tanzania
Ghana	Uganda
Guyana	Zambia

Figure C.4: Relationship between Net Public Foreign Assets and Growth Over Time, removing Outliers



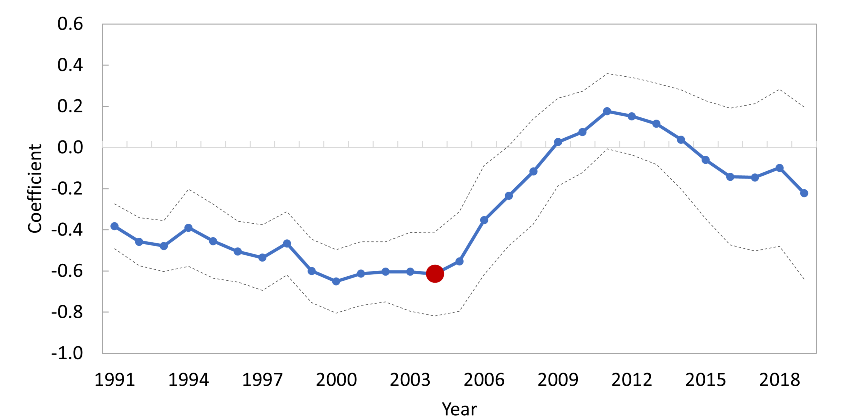
Notes: This figure plots the resulting coefficient β_T from equation (4) when outliers are removed using Cook's distance with the standard threshold $4/N$. The coefficient is the blue line, and the gray lines are a 90 percent confidence interval.

Figure C.5: Relationship between Net Public Foreign Assets and Growth Over Time, 20 year Rolling Regression



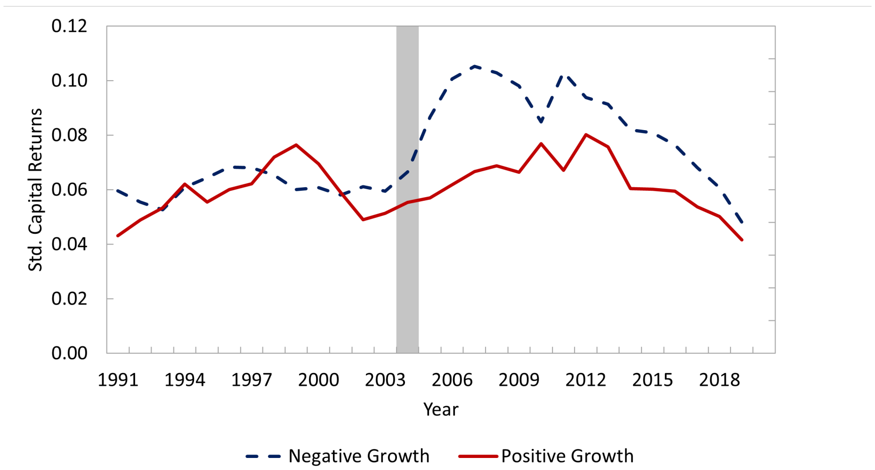
Notes: This figure plots the resulting coefficient β_T from equation (4) when the time period is kept constant at 20 years. That is, the reference year to calculate the net public foreign asset position of a country changes to maintain a 20 year window. The coefficient is the blue line, and the gray lines are a 90 percent confidence interval.

Figure C.6: Relationship between Net Public Foreign Assets and Growth Over Time, Geometric Mean



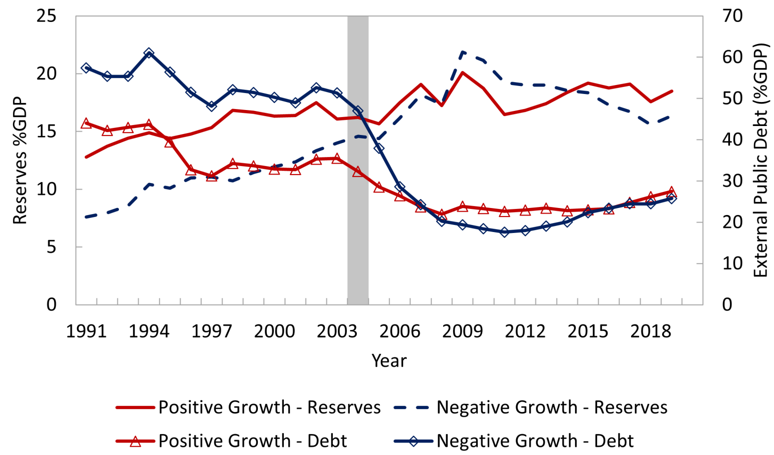
Notes: This figure plots the resulting coefficient β_T from equation (4) when growth rates are computed using geometric means instead of arithmetic means. The coefficient is the blue line, and the gray lines are a 90 percent confidence interval.

Figure C.7: Standard Deviation of Capital Returns (simple average)



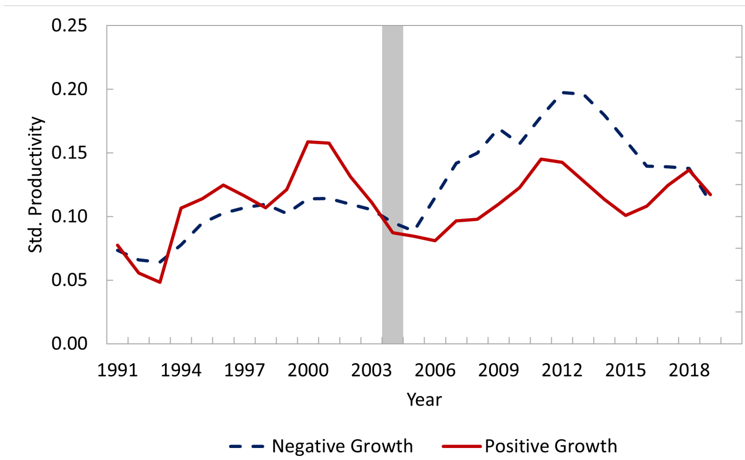
Source: Penn World Table and authors' calculations.

Figure C.8: International Reserves and Public External Debt to GDP (simple average) for Constant Sample



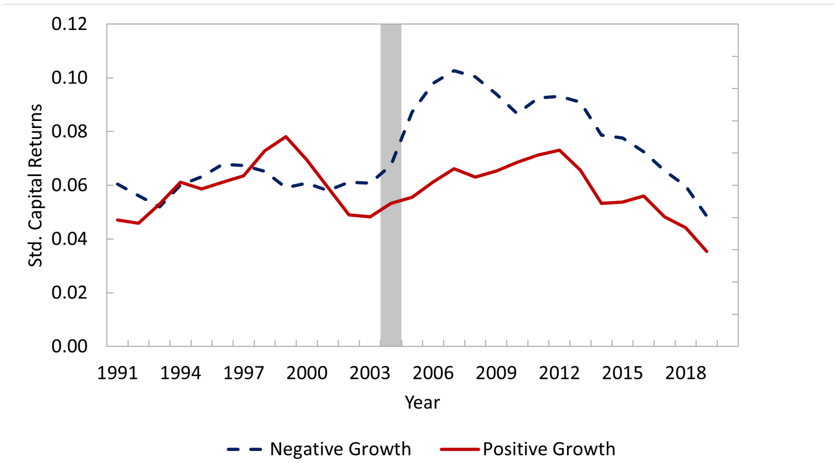
Source: IDS World Bank, WDI, World Bank, and BOP IMF, and authors' calculations.

Figure C.9: TFP Volatility (simple average) for Constant Sample



Source: Penn World Table and authors' calculations.

Figure C.10: Standard Deviation of Capital Returns (simple average) for Constant Sample



Source: Penn World Table and authors' calculations.

Appendix D Computational Algorithm

We begin by detrending all variables to ensure we are solving a stationary problem. An expanding rectangular grid is set over the continuous choice variables (K', D'). We approximate the AR(1) process for productivity shocks using the Rouwenhorst method.

The dynamic programming problem is solved using standard value function iteration beginning with an initial guess of $V_j(z, K, D) = 0$. j denotes the five country groupings for which the model is solved.

When solving for optimal policies we interpolate using cubic splines over next periods value function. We solve for policy functions using a modified quasi-Newton method to allow for rectangular box constraints.

To simulate moments from the model we take an arbitrary vector of parameters $\Theta = [\vec{\tau}_k, \vec{\tau}_s, \vec{\sigma}_z]$ and solve the model to obtain all decision rules. We simulate $N = 10,000$ economies for $T = 100,000$ periods.